Chapter 5 Image Restoration and Reconstruction

The principal goal of restoration techniques is to improve an image in some predefined sense.

Although there are areas of overlap, image enhancement is largely a subjective process, while restoration is for the most part an objective process.

Restoration attempts to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon. Thus, restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image.

The restoration approach usually involves formulating a criterion of goodness that will yield an optimal estimate of the desired result, while enhancement techniques are heuristic procedures to manipulate an image in order to take advantage of the human visual system.

Some restoration techniques are best formulated in the spatial domain, while others are better suited for the frequency domain.

5.1 A Model of the Image Degradation/Restoration Process

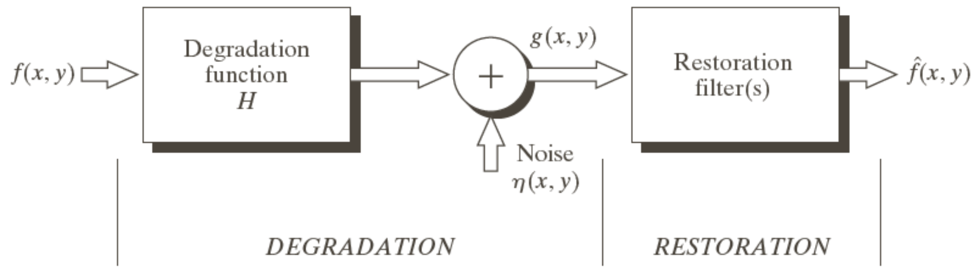


FIGURE 5.1 A model of the image degradation/restoration process.

Figure 5.1 shows an image degradation/restoration process.

The degraded image in the spatial domain is given by

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y) \quad (5.1\text{-}1)$$

where $h(x, y)$ is the spatial representation of the degradation function and "$\star$" indicates convolution. Therefore, we can have the frequency domain representation of (5.1-1)

$$G(u, v) = H(u, v)F(u, v) + N(u, v)\,. \quad (5.1\text{-}2)$$

These two equations are the bases for most of the restoration material in Chapter 5.

5.2 Noise Models

The principal sources of noise in digital images arise during image acquisition and/or transmission.

Spatial and Frequency Properties of Noise

In the spatial domain, we are interested in the parameters that define the spatial characteristics of noise, and whether the noise is correlated with the image.

Frequency properties refer to the frequency content of noise in the Fourier sense.

In general, we assume that noise is independent of spatial coordinates and it is uncorrelated with respect to the image itself.

Some Important Noise Probability Density Functions

Gaussian noise

Because of its mathematical tractability in both the spatial and frequency domains, Gaussian (normal) noise models are used frequently in practice.

The probability density function (PDF) of a Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2} \qquad (5.2\text{-}1)$$

where z represents intensity, $\bar{z}$ is the mean (average) value of z, and σ is its standard deviation.

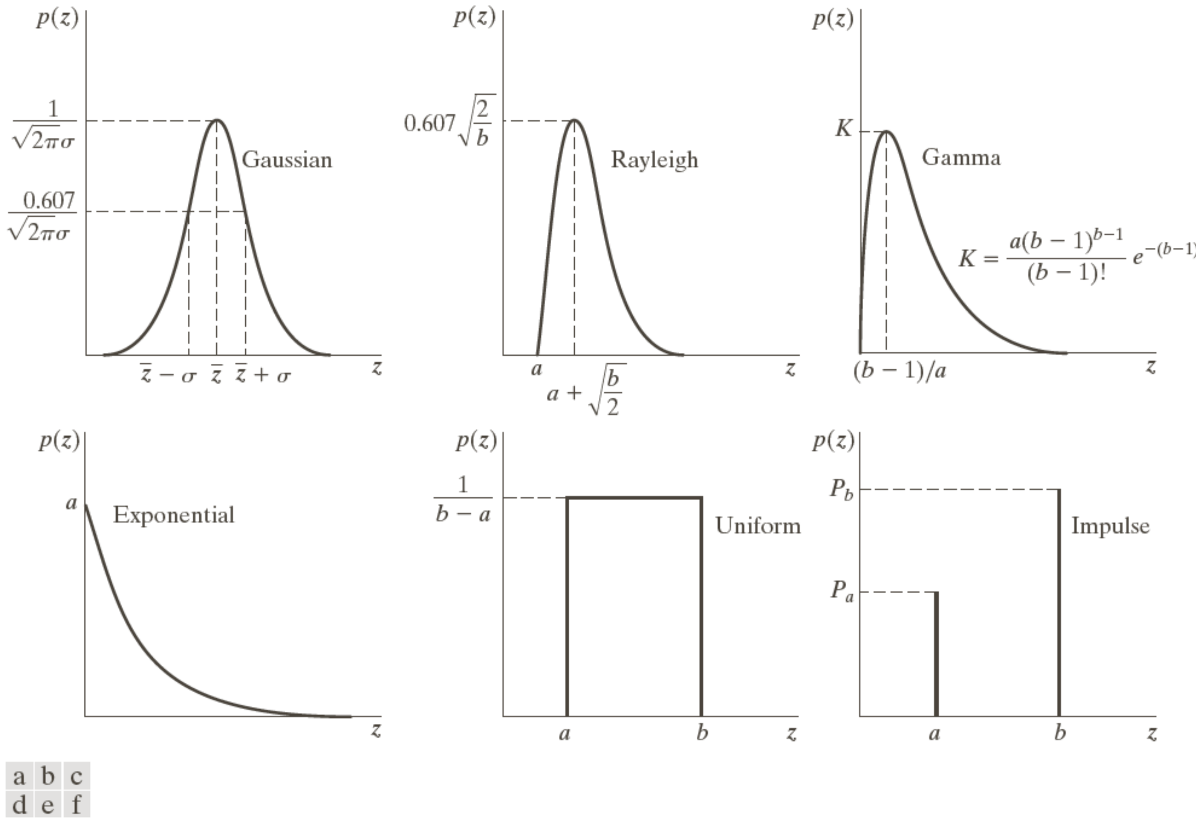


FIGURE 5.2 Some important probability density functions.

Rayleigh noise

The probability density function of Rayleigh noise is given by

$$p(z) = \begin{cases} \dfrac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \quad (5.2\text{-}2)$$

The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b/4} \quad (5.2\text{-}3)$$

and

$$\sigma^2 = \frac{b(4-\pi)}{4}. \quad (5.2\text{-}4)$$

Erlang (gamma) noise

The probability density function of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \tag{5.2-5}$$

where $a > 0$ and b is a positive integer. The mean and variance of this density are given by

$$\bar{z} = \frac{b}{a} \tag{5.2-6}$$

and

$$\sigma^2 = \frac{b}{a^2}\,. \tag{5.2-7}$$

Exponential noise

The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \tag{5.2-8}$$

where $a > 0$. The mean and variance of this density are given by

$$\bar{z} = \frac{1}{a} \tag{5.2-9}$$

and

$$\sigma^2 = \frac{1}{a^2}\,. \tag{5.2-10}$$

Uniform noise

The PDF of uniform noise is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le z \le b \\ 0 & \text{otherwise} \end{cases} \qquad (5.2\text{-}11)$$

The mean and variance of this density function are given by

$$\bar{z} = \frac{a+b}{2} \qquad (5.2\text{-}12)$$

and

$$\sigma^2 = \frac{(b-a)^2}{12}. \qquad (5.2\text{-}13)$$

Impulse (salt-and-pepper) noise

The PDF of impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \qquad (5.2\text{-}14)$$

If $b > a$, intensity b appears as a light dot in the image. Conversely, intensity a will appear like a dark dot.

If either P_a or P_b is zero, the impulse noise is called unipolar.

If neither P_a nor P_b is zero, and especially if they are approximately equal, the impulse noise values will resemble salt-and-pepper granules randomly distributed over the image.

Example 5.1: Noisy images and their histograms

Figure 5.3 shows a test pattern.

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Figure 5.4 shows the test pattern after addition of the six types of noise. Shown below each image is the histogram computed directly from that image.

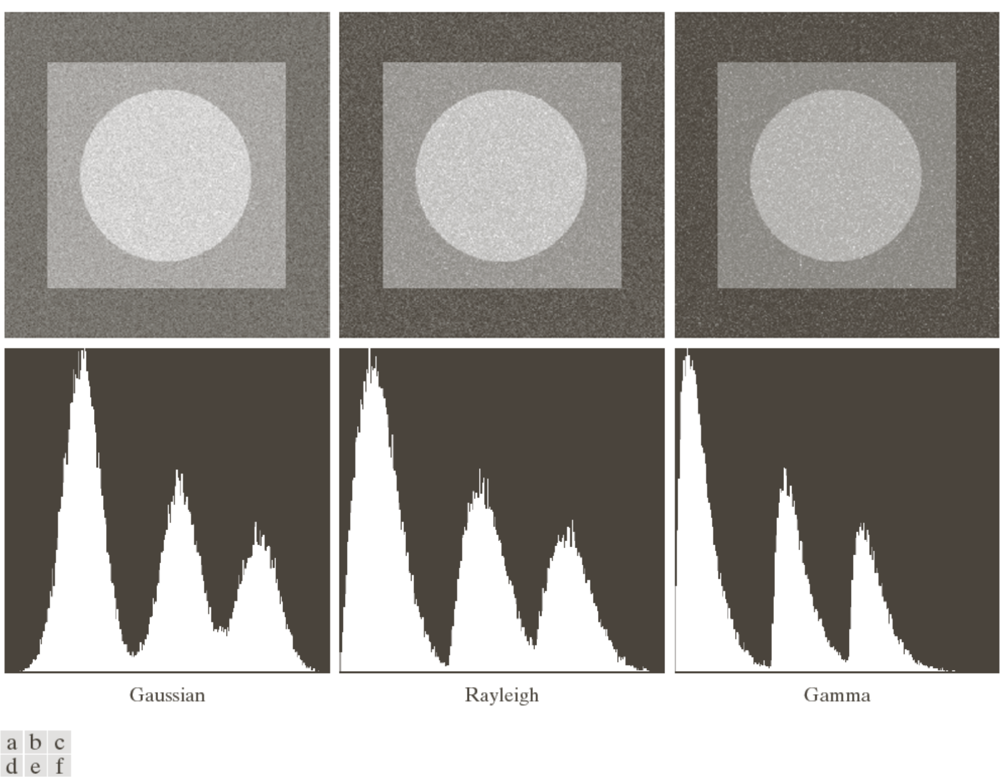


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

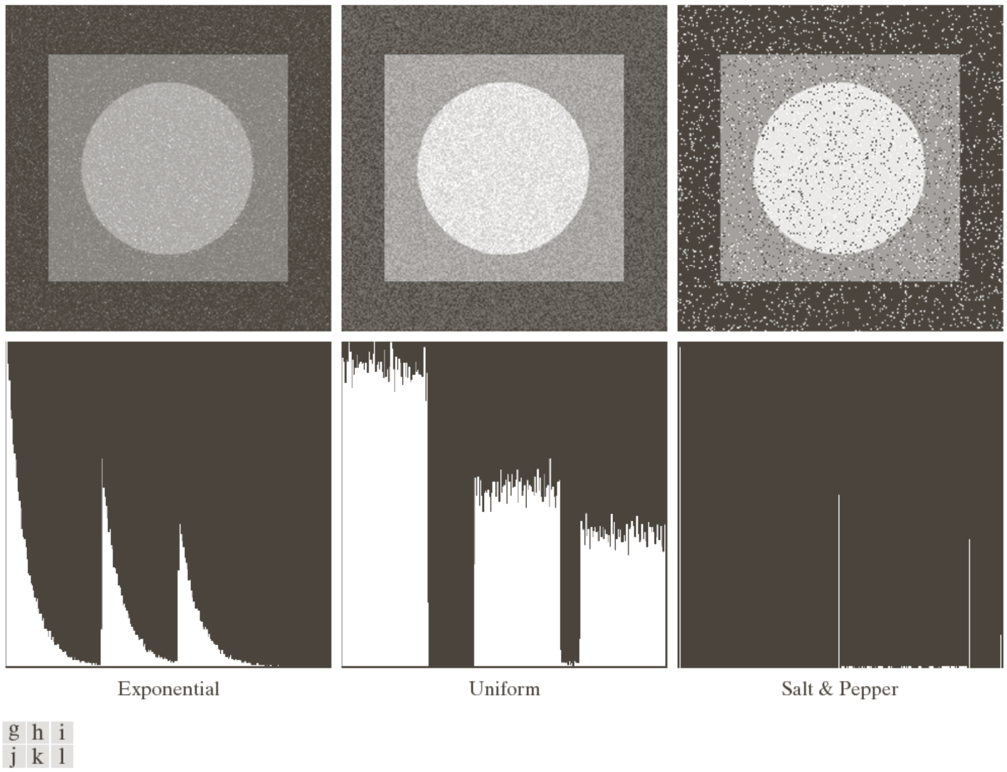


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Periodic Noise

Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition.

a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

The periodic noise can be reduced significantly via frequency domain filtering, which will be discussed in Section 5.4.

Estimation of Noise Parameters

The parameters of periodic noise can be estimated by inspection of the Fourier spectrum of the image.

Periodic noise tends to produce frequency spikes, which are detectable even by visual analysis.

In simplistic cases, it is also possible to infer the periodicity of noise components directly from the image.

Automated analysis is possible if the noise spikes are either exceptionally pronounced, or when knowledge is available about the general location of the frequency components of the interference.

It is often necessary to estimate the noise probability density functions for a particular imaging arrangement.

When images already generated by a sensor are available, it may be possible to estimate the parameters of the probability density functions from small patches of reasonably constant background intensity.

a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

The vertical stripes shown in Figure 5.6 were cropped from (a), (b), and (h) of Figure 5.4.

The histograms shown in Figure 5.6 were calculated using image data from these small stripes. We can see that the shapes of these histograms correspond closely to the shapes shown in (d), (e), and (k) of Figure 5.4.

The simplest use of the data from the image strips is for calculating the mean and variance of intensity levels. Let S denote a stripe and $p_S(z_i)$, $i = 0, 1, 2, \ldots, L-1$, denote the probability estimates of the intensities of the pixels in S, then the mean and variance of the pixels in S are

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_S(z_i) \qquad (5.2\text{-}15)$$

and

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_S(z_i)\,. \qquad (5.2\text{-}16)$$

The shape of the histogram identifies the closest probability density function match.

The Gaussian probability density function is completely specified by these two parameters.

For the other shapes discussed previously, we can use the mean and variance to solve the parameters a and b.

Impulse noise is handled differently because the estimate needed is of the actual probability of occurrence of the white and black pixels.

5.3 Restoration in the Presence of Noise Only – Spatial Filtering

When the only degradation present in an image is noise,

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y) \tag{5.1-1}$$

and

$$G(u, v) = H(u, v)F(u, v) + N(u, v) \tag{5.1-2}$$

become

$$g(x, y) = f(x, y) + \eta(x, y) \tag{5.3-1}$$

and

$$G(u, v) = F(u, v) + N(u, v)\,. \tag{5.3-2}$$

Since the noise terms are unknown, subtracting them from $g(x, y)$ or $G(u, v)$ is not a realistic option.

In the case of periodic noise, it usually is possible to estimate $N(u, v)$ from the spectrum of $G(u, v)$.

Mean Filters

Arithmetic mean filter

Let S_{xy} represent the set of coordinates in a subimage window of size $m \times n$, centered at (x, y). The arithmetic mean filter computes the average value of the corrupted image $g(x, y)$ in S_{xy}. The value of the restored image $\hat{f}$ at point (x, y) is the arithmetic mean computed in the region S_{xy}:

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)\,. \tag{5.3-3}$$

Geometric mean filter

Using a geometric mean filter, an image is restored by

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} \mathrm{g}(s,t) \right]^{\frac{1}{mn}}. \tag{5.3-4}$$

A geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process.

Harmonic mean filter

The harmonic mean filter is given by the expression

$$\hat{f}(x,y) = \frac{mn}{\sum\limits_{(s,t)\in S_{xy}} \frac{1}{\mathrm{g}(s,t)}}, \tag{5.3-5}$$

which works well for some types of noise like Gaussian noise and salt noise, but fails for pepper noise.

Contraharmonic mean filter

The contraharmonic mean filter yields a restored image based on the expression

$$\hat{f}(x,y) = \frac{\sum\limits_{(s,t)\in S_{xy}} \mathrm{g}(s,t)^{Q+1}}{\sum\limits_{(s,t)\in S_{xy}} \mathrm{g}(s,t)^{Q}}, \tag{5.3-6}$$

where Q is called the order of the filter.

The contraharmonic mean filter is well suited for reducing or eliminating the effects of salt-and-pepper noise.

For positive values of Q , it eliminates pepper noise.

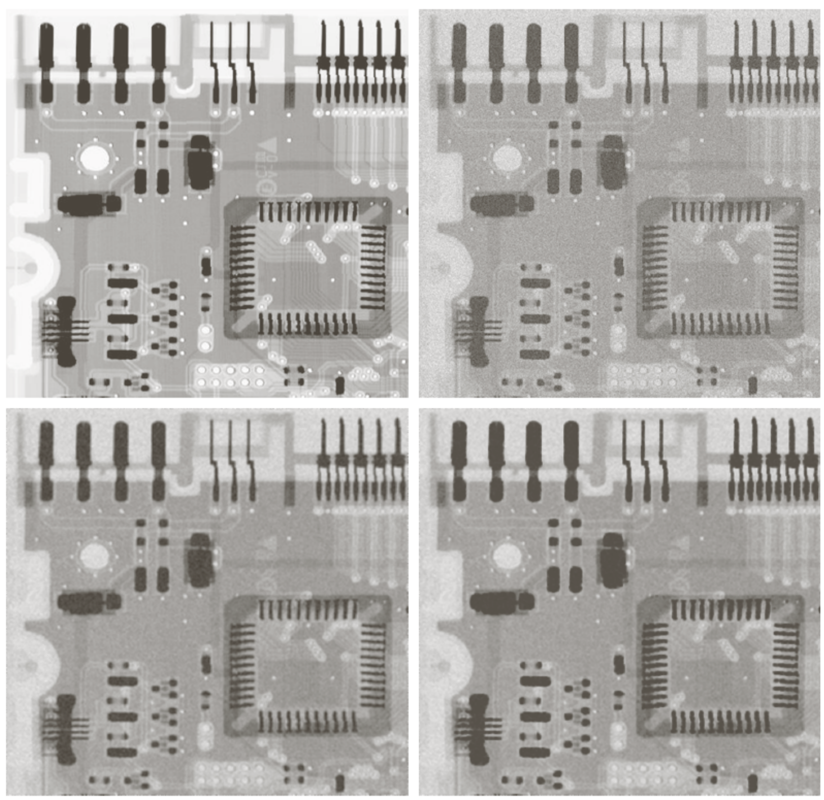
For negative values of Q , it eliminates salt noise.

When $Q = 0$, the contraharmonic mean filter reduces to the arithmetic mean filter.

When $Q = -1$, the contraharmonic mean filter becomes the harmonic mean filter.

Example 5.2: Illustration of mean filters

Figure 5.7 (a) shows an 8-bit image, and Figure 5.7 (b) shows its corrupted version with additive Gaussian noise of zero mean and variance of 400.

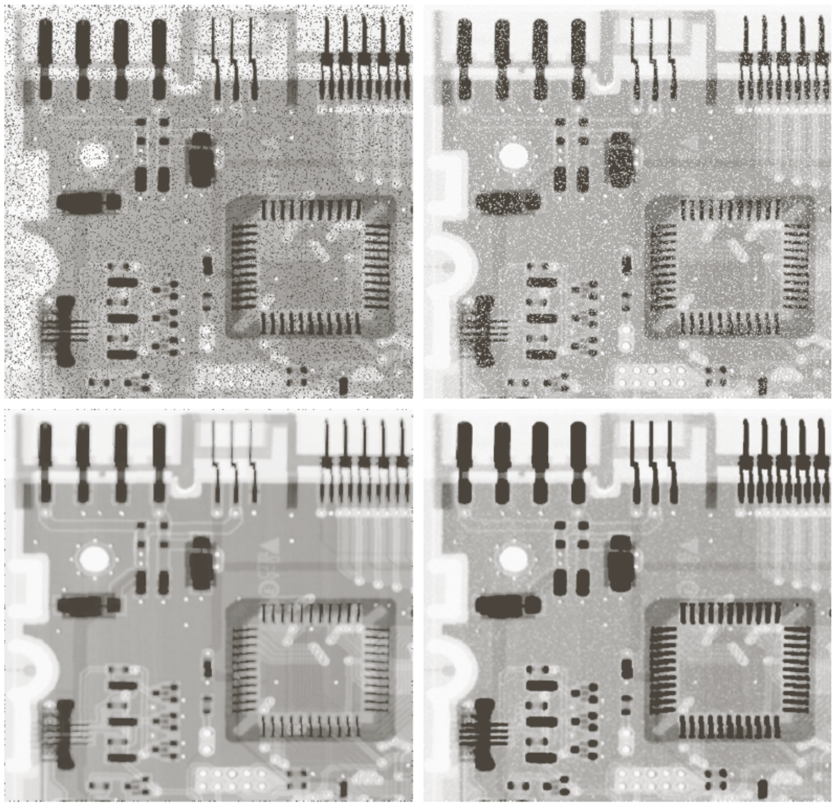


a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Figure 5.7 (c) and Figure 5.7 (d) show the result of filtering the noisy image with a 3×3 arithmetic mean filter and a 3×3 geometric mean filter, respectively.

Figure 5.8 (a) and Figure 5.8 (b) show the images corrupted by 10% pepper noise and 10% salt noise, respectively.



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Figure 5.8 (c) shows the result of filtering Figure 5.8 (a) using a contraharmonic mean filter with $Q = 1.5$.

Figure 5.8 (d) shows the result of filtering Figure 5.8 (b) using a contraharmonic mean filter with $Q = -1.5$.

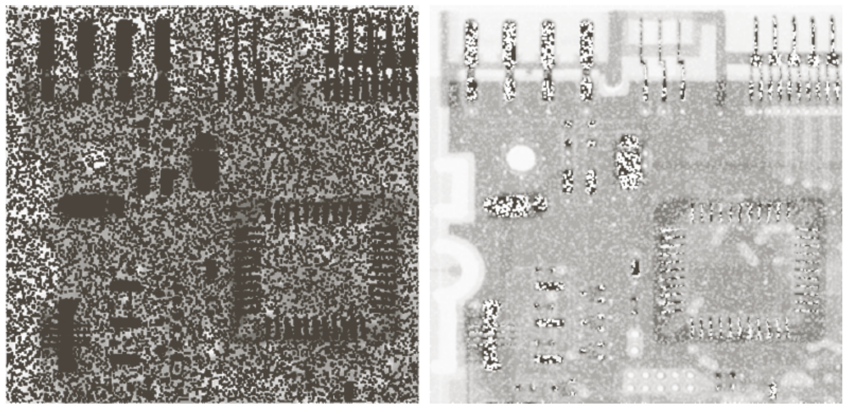
The positive-order filter did a better job of cleaning the background, at the expense of slightly thinning and blurring the dark areas.

The opposite was true of the negative-order filter.

In general, the arithmetic and geometric mean filters are suited for random noise like Gaussian or uniform noise.

The contraharmonic mean filter is well suited for impulse noise, with the disadvantage that it must known whether the noise is dark or light in order to select Q.

Figure 5.9 shows some results of choosing the wrong sign for Q.



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Order-Statistic Filters

As discussed in Chapter 3, order-statistic filters are spatial filters whose response is based on ordering (ranking) the values of the pixels contained in the image area encompassed by the filter.

Median filter

The best-known order-statistic filter is the median filter, which will replace the value of a pixel by the median of the intensity levels in the neighbourhood of that pixel:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{\mathrm{g}(s,t)\}\,. \tag{5.3-7}$$

For certain types of random noise, the median filters can provide excellent noise-reduction capabilities.

The median filters are particularly effective in the presence of both bipolar and unipolar impulse noise.

Max and min filters

The max and min filters are defined as

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{\mathrm{g}(s,t)\} \tag{5.3-8}$$

and

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{\mathrm{g}(s,t)\}\,. \tag{5.3-9}$$

The max filter is useful for finding the brightest points in an image, while the min filter can be used for finding the darkest points in an image.

Midpoint filter

The midpoint filter computes the midpoint between the maximum and minimum values in the area encompassed by the filter:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\}\right]. \qquad (5.3\text{-}10)$$

The midpoint filter works best for random distributed noise, like Gaussian or uniform noise.

Alpha-trimmed mean filter

Suppose that we delete the $d/2$ lowest and the $d/2$ highest intensity values of $g(s,t)$ in S_{xy}. Let $g_r(s,t)$ represent the remaining $mn-d$ pixels, an alpha-trimmed mean filter is given by

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t). \qquad (5.3\text{-}11)$$

When $d=0$, the alpha-trimmed mean filter is reduced to the arithmetic mean filter.

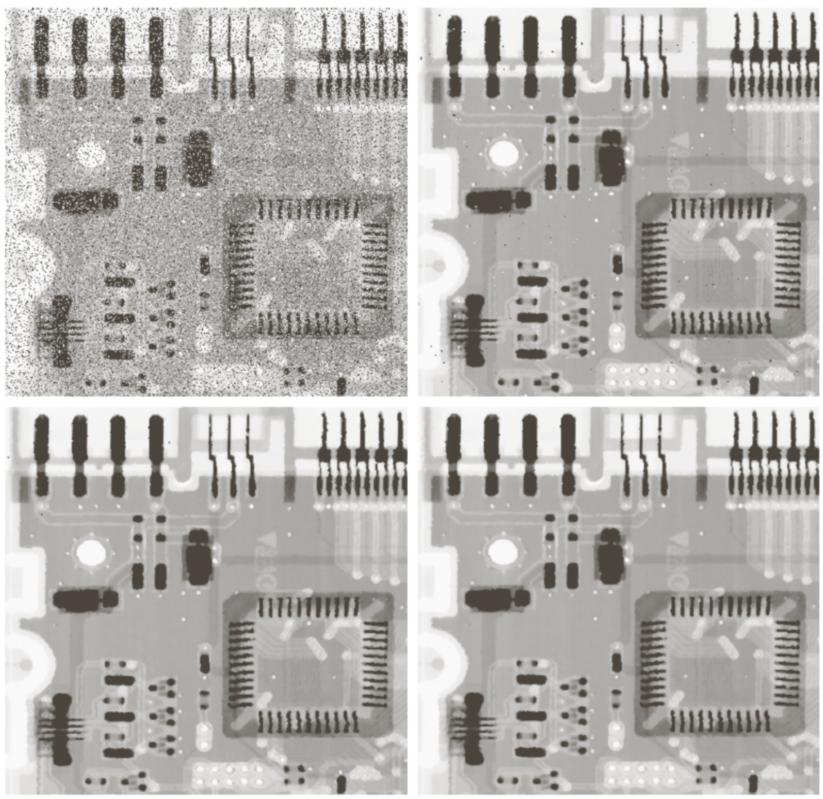
If $d=mn-1$, the alpha-trimmed mean filter becomes a median filter.

Example 5.3: Illustration of order-statistic filters

Figure 5.10 (a) shows the image corrupted by salt-and pepper noise with probabilities $P_a = P_b = 0.1$.

Figure 5.10 (b) shows the result of median filtering with a filter of size 3×3.

Figure 5.10 (c) and Figure 5.10 (d) show the result of applying the same filter on Figure 5.10 (b) and Figure 5.10 (c), respectively.



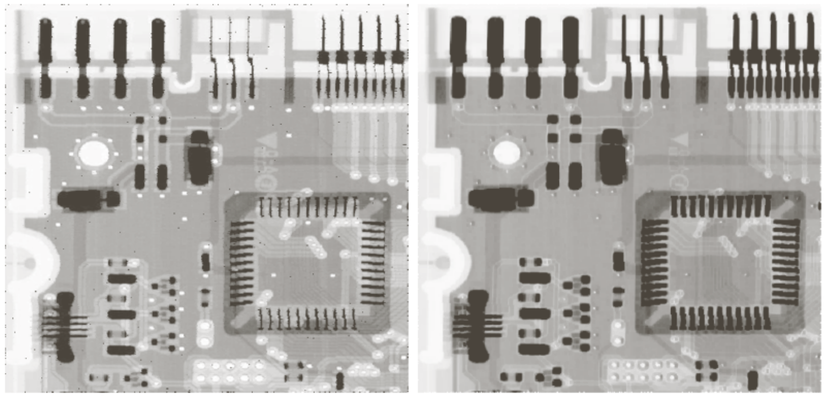
a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Figure 5.11 (a) shows the result of applying the max filter to the pepper noise image of Figure 5.8 (a).

Figure 5.11 (b) shows the result of applying the min filter to the image of Figure 5.8 (b).

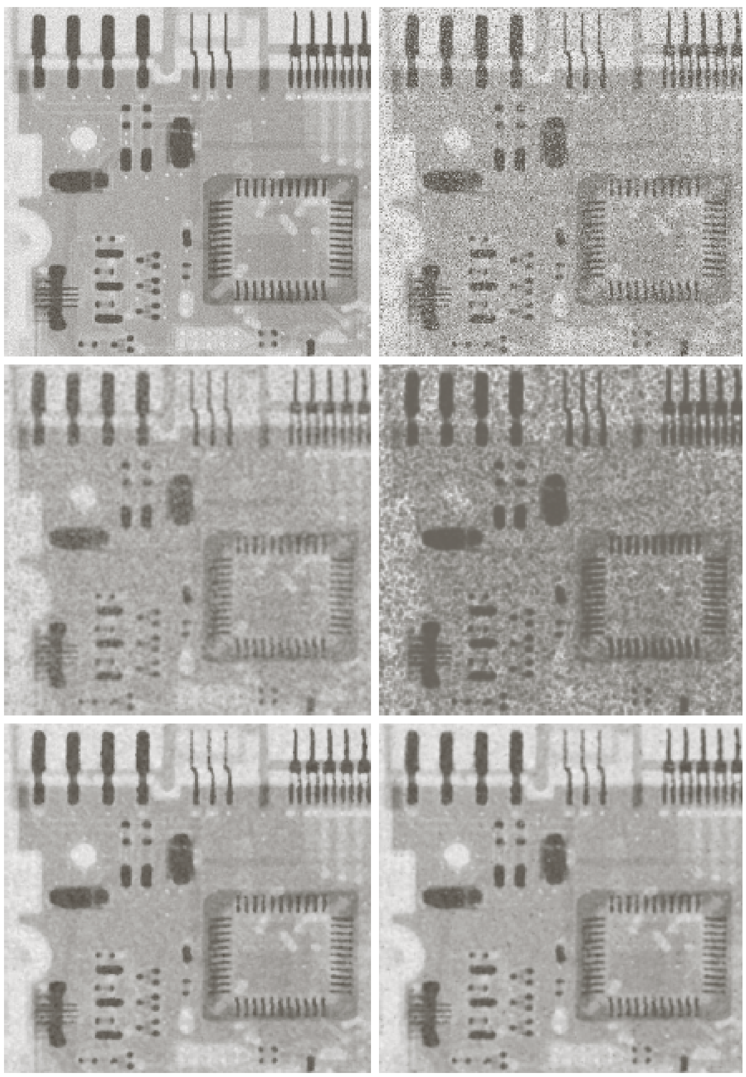
The min filter did a better job on noise removal, but it removes some white points around the border of light objects.



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.

The results of applying the alpha-trimmed filter are shown in Figure 5.12.



a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

In Figure 5.12 (e), it should be $d = 6$.