The Effects of Employer Matching and Income Risk in 401(k) Plans*

Jie Zhou †

Division of Economics, Nanyang Technological University, Singapore

*I thank Audra Bowlus, John Burbidge, James Davies, Igor Livshits, Jim MacGee, Stephen Sapp, and two anonymous referees for their helpful comments.

†Mailing address: Division of Economics, School of Humanities and Social Sciences, Nanyang Technological University, Singapore 639798. Tel: +65 65138162. E-mail: zhoujie@ntu.edu.sg.

The Effects of Employer Matching and Income Risk in 401(k) Plans

Abstract

Using a simple life-cycle model, this paper studies the effects of employer matching and income risk on employees' 401(k) participation and contributions. We find that both employer matching and income risk have a large impact on employee decisions. The effects differ for different income and age groups.

JEL Classifications: G23; J32

Keywords: 401(k) plan; Employer matching; Income risk

1 Introduction

Retirement saving has changed dramatically over the last two decades in the United States. There has been a shift from employer-managed defined benefit (DB) pension plans to defined contribution (DC) pension plans that are largely controlled by employees.[1] In contrast to DB plans, participation in DC plans often requires active decisions by eligible employees. These employees need to make decisions about whether or not to participate, how much to contribute (subject to plan and legislative limits), and how to invest their money. Employee contributions and their returns are tax-deferred until withdrawal from the plans. Employers often provide matching contributions (up to a pre-determined limit) for employee contributions.[2] This is typically the case for a 401(k) plan.

Given that employers usually match employee contributions in 401(k) plans, a number of empirical studies have examined the effects of employer matching on employees' participation and contribution decisions.[3] The literature commonly suggests that 401(k) participation rate increases in the presence of employer matching, although the estimated magnitude varies. For the effects of employer matching on employee contributions, the empirical results are mixed. On the one hand, Andrews (1992), EBRI (1994), and Huberman

[1]As of 2003, DC plan assets stood at $2.4 trillion, and DB plan assets were estimated at $2.1 trillion. There were more participants in DC plans than in DB plans. See Vanguard Group (2004).

[2]Mitchell et al. (2005) find that 82% of employers provide matching for employee contributions in their sample of 507 plans. Even and Macpherson (2005) show that in the April 1993 Current Population Survey (CPS), 81% of employees eligible for 401(k) plans are offered employer matching.

[3]These studies use plan data as well as survey data. For plan data, see Papke (1995), Papke and Poterba (1995), USGAO (1997), Clark and Schieber (1998), Kusko et al. (1998), Choi et al. (2004), Mitchell et al. (2005), and Huberman et al. (2007). For survey data, see Andrews (1992), Bassett et al. (1998), Munnell et al. (2002), Papke (2004a, b), Even and Macpherson (2005), and Engelhardt and Kumar (2007).

et al. (2007) find that the presence of employer matching has a negative effect on employee contributions. Munnell et al. (2002) also show that employees tend to contribute less when facing a higher employer match rate. On the other hand, Papke (1995) finds a positive effect of employer matching on employee contributions at low match rates but a negative effect at higher match rates. Kusko et al. (1998) find little evidence that employees respond to increases in the employer match rate by reducing their own contribution rates. Clark and Schieber (1998) and Engelhardt and Kumar (2007) show that the presence of employer matching has a positive effect on employee contributions, and a higher matching rate induces a higher employee contribution rate. The mixed results may be because in theory, the effects of employer matching are less straightforward. Employer matching has both an income effect and a substitution effect. Whether the matching encourages employees' own contributions or not depends on which effect dominates. The other reason is that the literature uses different data sets. One observation from the empirical studies is that employee income tends to be lower for those studies that find a positive impact of employer matching on employee contributions, while employee income tends to be higher for those studies that find a negative impact.

Although many studies have focused on what determines 401(k) participation and contributions and the role of employer matching, less attention has been paid on the effects of income risk in 401(k) plans when there is employer matching. We address this issue in the paper by developing a simple 3-period model and numerically solving for 401(k) participation and contribution decisions for eligible employees that face stochastic labor income, borrowing constraints, a progressive tax system, and different tax treatments for taxable and tax-deferred accounts.[4] Employees in the model form rational expectations about labor

[4]For the purposes of this study we use "tax-deferred account" to mean 401(k) plans.

income, pension benefits, and social security benefits.[5] We have the following key findings.

First, a typical employer matching program dramatically increases the 401(k) participation rate for young employees, but not for old employees. The effect is strongest for low-income young employees. If we look at employees that participate in 401(k) plans no matter whether there is employer matching or not, we find that the average contribution rates of young employees in all income levels increase when there is employer matching. In period 2 (when employees are old), however, the average contribution rates are lower for all income levels when there is employer matching. This is likely because employee contribution rates are high in period 2 even when there is no employer matching. Employer matching only provides a limited incentive when employees are old, and the income effect dominates the substitution effect when there is employer matching.

Second, retirement wealth increases under employer matching for employees that participate in 401(k) plans in the model. Moreover, low-income employees enjoy a higher increase (in percentage terms) in retirement wealth than high-income employees.

Third, the model indicates that income risk has a large impact on 401(k) participation and contributions. This is because: (i) income risk affects the precautionary saving motive, and (ii) it also affects the expected replacement rate for social security benefits and hence retirement saving behavior. We find that a higher variance of persistent income shocks lowers the participation rate and contribution rate for low-income employees, while it increases employee contributions for high-income employees. A higher variance of transitory income shocks lowers the participation rates for all income levels. For employee contributions, it increases the contribution rate of young employees but lowers the contribution rate of old

[5] We do not study behavioral influences here. For those influences, please see Choi et al. (2004) and other papers cited in Bailey et al. (2003).

employees. We also consider a possible disastrous labor income draw. We show that it increases the participation rate of low- and middle-income young employees considerably, while its effect on employee contributions is small.

Forth, a higher contribution limit has a small effect on 401(k) participation rate. An increase in the contribution limit also has a small effect on contribution rates of young employees, while it has a large effect on contribution rates of old employees.

Previous work has also highlighted the importance of other plan features in affecting 401(k) plan participation and contribution rates. These features include: plan liquidity (access to pension funds through withdrawal or borrowing), enrollment protocol, participant control over assets, and the amount and quality of plan and investment information provided by the plan sponsor.[6] The availability of a DB plan could also be relevant to 401(k) plan participation and contributions.[7] The effects of social and psychological influences have also been studied in the literature.[8] However, we abstract from these other features and focus on the effects of employer matching in this paper.

The paper is organized as follows. Section 2 describes the model's assumptions and set-up. Section 3 discusses the parameterization of the model. Sections 4 presents the simulation results of our benchmark and sensitivity analysis. Section 5 concludes. For the numerical procedure used to solve the model, see Appendix.

---

[6]For pension plan liquidity, see USGAO (1997), Munnell et al. (2002), and Mitchell et al. (2005); for enrollment protocol, see Madrian and Shea (2001), Vanguard (2001), Choi et al. (2004), and Choi, Laibson et al. (2004); for participant control over assets, see Papke (2004a, b); for the amount and quality of plan and investment information, see Clark and Schieber (1998).

[7]See Clark and Schieber (1998), Munnell et al. (2002), Even and Macpherson (2005), and Huberman et al. (2007).

[8]See Bailey et al. (2003) for a survey.

2 Model

We consider a 3-period model where employees maximize discounted utility from consumption. There is a progressive income tax code. Employees work in the first two periods and face idiosyncratic labor income shocks. They are retired in the third period and receive their retirement income. Employees have access to both a regular taxable account and a 401(k) type pension plan (tax-deferred account).[9] We study their 401(k) participation and contribution decisions with and without employer matching.

2.1 Preferences

Time is discrete and j denotes model period. Each employee has preferences defined over a consumption stream. Preferences are represented by

$$E_1 \sum_{j=1}^{3} \beta^{j-1} \frac{C_j^{1-\gamma}}{1-\gamma} \tag{1}$$

where $\beta < 1$ is the discount factor, γ is the coefficient of relative risk aversion, and C_j denotes consumption in period j.

2.2 Labor Income Process

In each working period $1 \leq j \leq 2$, employees receive a stochastic endowment. Following the standard specification in the life-cycle literature, we consider both persistent and transitory income shocks.[10] The income of employee i in period j, Y_{ij}, is exogenously given by:

$$log(Y_{ij}) = \bar{y}_j + z_{ij} + u_{ij} \tag{2}$$

[9] This means all employees are eligible for 401(k) plans in the model.

[10] See Carroll (1992), Carroll and Samwick (1997), Hubbard et al. (1994, 1995), Huggett and Ventura (2000), Gourinchas and Parker (2002), and Gomes and Michaelides (2005).

where $\bar{y}_j$ is the mean log income of all period j employees; the transitory shocks, u_{ij}, are independent and identically normally distributed $N(0, \sigma_u^2)$; and the persistent shocks, z_{ij}, follow an AR(1) process:

$$z_{ij} = \rho z_{ij-1} + \xi_{ij} \tag{3}$$

where ξ_{ij} are independent and identically normally distributed $N(0, \sigma_\xi^2)$ and are uncorrelated with u_{ij}. We also assume $\bar{y}_j = log(G_j) + \bar{y}_{j-1}$, where G_j governs the age-profile of $\bar{y}_j$.

When $j = 3$, the employee is retired. Retirement income is given by:

$$log(Y_{i3}) = log(\lambda_i(z_{i2})) + \bar{y} + z_{i2} \tag{4}$$

where $\bar{y}$ is the mean of $\bar{y}_1$ and $\bar{y}_2$, and the replacement rate (λ_i) depends on employee i's persistent income shock in period 2 (z_{i2}).[11] This specification simplifies the solution of the model because we do not need to track the employee's entire income history.

2.3 Financial Assets, Accounts, and Taxation

Employees can hold risk-free financial assets. The constant risk-free return is r. Financial assets can be accumulated in two accounts: a regular taxable account (TA) and a tax-deferred account (TDA). In the TA, all taxes are paid on an on-going basis. Labor income and interest income are taxed at the rate of τ. The TDA defers tax payments on contributions and returns. Throughout working life $j \leq 2$, each eligible employee can contribute to the TDA up to a fraction $\bar{q}$ of before-tax labor income. Employers may match employees' contributions. During the retirement period, contributions to the TDA are not

[11] Retirement income is modeled as a fraction (the replacement rate) of lifetime average earnings, where lifetime average earnings depend on an employee's persistent income shock in period 2.

allowed and the employee withdraws funds from the TDA. The employee pays tax on the withdrawals at income tax rate, τ. Assets in the TDA can also be accessed prior to retirement (period 3) at the cost of a penalty rate ϕ in addition to income taxes. We assume that borrowing is not allowed in the model economy.

We incorporate a progressive income tax code in the model, which means τ depends on the employee's income level. The income tax code is comprised of a number of brackets, defined by different thresholds with corresponding different marginal tax rates. Each employee's income subject to taxation is defined to be the sum of labor income (net of contributions), interest income, and withdrawals (there is an additional penalty for early withdrawals). More details on the tax code are provided in the parameterization section.

2.4 Wealth Dynamics and Employees' Optimization Problem

At the beginning of each working period, income shocks are realized and employees receive their labor income in the TA. Then they choose current contributions to (or withdrawals from) the TDA and current consumption. If employees make contributions, they may also receive employer matching contributions in the TDA. For employee i, let W_{ij}^T be the after-tax financial wealth in the TA plus current labor income at the beginning of period j (before current contributions and consumption). Similarly, W_{ij}^D is the wealth in the TDA at the beginning of period j (before current contributions and employer matching). The wealth dynamics are given by (we drop i here):

$$W_1^T = Y_1 \tag{5}$$

$$W_1^D = 0 \tag{6}$$

For $j = 1$ or 2,

$$W_{j+1}^T = [1 + r(1 - \tau_{j+1})][W_j^T - q_j Y_j - (1 - q_j) Y_j \tau_j + X_t(1 - \tau_j - \phi) - C_j] + Y_{j+1} \quad (7)$$

$$W_{j+1}^D = (1 + r)(W_j^D + q_j Y_j + M_j - X_j) \quad (8)$$

where the term in the first bracket of equation (7) is the after-tax gross return on the financial assets held in the TA from period j to period $j + 1$, Y_j is the labor income in period j, $q_j \in [0, \bar{q}]$ is the contribution rate, $X_j \in [0, W_j^D]$ is the amount of withdrawal from the TDA, and C_j is consumption.[12] The term in the first bracket of equation (8) denotes the gross return on the assets held in the TDA from period j to period $j+1$, and M_j is the amount of employer matching, which is a function of the employee's contribution amount $(q_j Y_j)$.

We impose the following borrowing constraints for all j:

$$W_j^T \geq 0, \; W_j^D \geq 0 \quad (9)$$

The problem the employee faces is to maximize (1) subject to constraints given by (5) to (9), and to the labor income process given by (2) to (4), in addition to the non-negativity constraint on withdrawals and consumption.

The control variables are: contribution rate (q_j), withdrawal (X_j), and consumption (C_j). There are three state variables in each period: the wealth level in the TA (W_j^T), the wealth level in the TDA (W_j^D), and persistent income shocks (z_j). The Bellman equation for this problem is given by:

$$V_j(W_j^T, W_j^D, z_j) = \max_{q_j, X_j, C_j} \frac{C_j^{1-\gamma}}{1-\gamma} + \beta E_j \left[V_{j+1}(W_{j+1}^T, W_{j+1}^D, z_{j+1}) \right] \quad (10)$$

[12] τ_j represents a progressive income tax code.

Given the finite nature of the problem, a solution exists and can be obtained by backward induction. For details see the numerical solution in Appendix.

3 Parameterization

Each model period represents 20 years. The constant risk-free rate r is set at 80%, which implies a 3% compound yearly risk-free rate. We choose the discount factor β equal to 0.5556.[13] The coefficient of relative risk aversion γ is set at 2.

3.1 Labor Income Process

For the labor income process, first we need to specify the median income of employees in period 1 and the age-earnings profile. Recall that $\bar{y}_1$ is the mean log income of all period 1 employees. Let $\bar{Y}_1 = exp(\bar{y}_1)$.[14] Thus, $\bar{Y}_1$ is the median income of all period 1 employees in the model and is set to be \$40,000.[15] G_j reflects the age-earnings profile ($\bar{y}_1, \bar{y}_2$ or $\bar{Y}_1, \bar{Y}_2$). We set $G_2 = 1.41$.[16]

The remaining parameters of the labor income process in working periods are ρ, σ_ξ^2, and σ_u^2. The literature estimates a high value of ρ (close to 1) using yearly data. See Hubbard et al. (1994, 1995), Huggett and Ventura (2000), Gourinchas and Parker (2002), and Storesletten et al. (2004). We choose the value of 0.995, which implies that ρ is $0.995^{20} = 0.9046$ for a 20-year period. For the persistent shocks, we choose $\sigma_\xi^2 = 0.02$, which

[13] $\beta = \frac{1}{1+r} \approx 0.5556$.

[14] If income is log normally distributed, the mean log income and the median income are related as follows: median income = exp (mean log income).

[15] This number is the median income of all households with heads aged 21 to 40 in the 2004 Survey of Consumer Finances (SCF).

[16] This is the ratio of median income for households with heads aged 41-60 to that of households with heads aged 21-40 in the 2004 SCF.

implies that a one standard deviation shock increases or decreases earnings by about 15%. For the transitory shocks, we let $\sigma_u^2 = 0.02$. Both variances are in the range of estimates in Hubbard et al. (1994). We discretize the idiosyncratic income shocks using Tauchen method outlined in Adda and Cooper (2003). A Markov process with three states (characterized by a transition matrix) is used to approximate the first-order auto-regression.[17]

3.2 Social Security Benefits

During the retirement period (period 3), employees receive social security benefits. Employees with different working-life average earnings have different replacement rates in the U.S. social security system. For computational tractability, we let the working-life average earnings depend on employees' persistent income levels in period 2. We set the replacement rates according to the U.S. social security benefit formula (with lower income employees having a higher replacement rate).[18]

3.3 Contribution Limit, Employer Matching, and Withdrawal

According to Joulfaian and Richardson (2001), for employees participating in DC plans, the average employee contribution rate is 5.9%. Approximately 85% contribute less than 10%. We set the contribution limit, $\bar{q}$, to be 10% before retirement. We will examine the effect of contribution limit by increasing the limit in section 4.4. Employee contributions are not allowed during the retirement period. Employers often provide matching contributions for employee contributions. We consider two cases: (1) no match; and (2) a typical employer matching formula: employers match 50% of first 6% of employee contributions. We set the

[17] The three persistent income levels in period 1 are $27,850, $40,000, and $57,440, respectively.

[18] For the U.S. social security benefit formula, see Figure 1 in Huggett and Parra (2006).

early withdrawal penalty (before period 3), ϕ, at 10% . This penalty is a common feature of many tax-deferred retirement accounts in the United States.

Table 1 reports the parameter values we use.

Parameter	Parameter Value
Discount Factor (β)	0.5556
Risk Aversion (γ)	2
Risk-free Rate (r)	0.80
Age-Earnings Profile (G_2)	1.41
AR(1) Term (ρ)	0.9046
Variance of Transitory Shocks (σ_u^2)	0.02
Variance of Persistent Shocks (σ_ξ^2)	0.02
Contribution Limit ($\bar{q}$)	10%
Early Withdrawal Penalty (ϕ)	10%

Table 1: Parameter values

3.4 Tax Code

For the income tax, our strategy is to mimic the federal income tax code in the United States prevailing before President Bush's tax cut of 2001.

There are 5 tax brackets, with marginal tax rates of 15%, 28%, 31%, 36%, and 39.6%. We set the taxable income thresholds at $40,000, $100,000, $150,000, and $260,000 respectively, which were roughly the thresholds during 1993-2000. In order to find the corresponding tax brackets in terms of total income, we need to approximate the complex exemptions and deductions present in the actual tax code first. We let the sum of the standard deduction and personal exemptions be \$12,000, which is 30% of $\bar{Y}_1$. We normalize $\bar{Y}_1$ as 1. Table 2 describes the marginal tax rates we use:

Income	Normalized Income	Marginal Tax Rate
(\$0, \$12,000]	(0.00, 0.30]	0%
(\$12,000, \$52,000]	(0.30, 1.30]	15%
(\$52,000, \$112,000]	(1.30, 2.80]	28%
(\$112,000, \$162,000]	(2.80, 4.05]	31%
(\$162,000, \$272,000]	(4.05, 6.80]	36%
\$272,000 +	6.80 +	39.60%

Table 2: Tax code used in the model

4 Simulation Results

In this section we present and discuss our simulation results. We conduct experiments with and without employer matching. Then we perform sensitivity analysis. For each experiment, we report 401(k) plan (TDA) participation rates by persistent income level for eligible employees and the average contribution rates by persistent income level for 401(k) plan participants.

4.1 Benchmark

Table 3 shows our simulation results when there is no employer matching. The 401(k) participation rate in period 1 (when employees are young) for eligible employees increases with employee persistent income. For eligible employees with low persistent income, only 28.9% choose to participate in 401(k) plans in the first period. This proportion increases to

48.8% and 86.4% for eligible employees with middle and high persistent income, respectively. The 401(k) plan participation rates for eligible employees are much higher in period 2 compared to those in period 1 for all income levels. Almost all eligible employees choose to participate in 401(k) plans in period 2. During the retirement period, no contributions are allowed and employees withdraw funds from the TDA. Thus, both the participation rate and contribution rate are zero in period 3. Empirical studies have also found that the 401(k) participation rate is positively related to employees' income and age.[19] We find similar results in our simulations. However, our model predicts much higher participation rate for old employees. This is likely because all uncertainties are realized in period 2 and employees want to take advantage of the tax-deferral feature.

For 401(k) plan participants, the contribution rate varies with persistent income as well as period. In period 1, the contribution rate increases with persistent income. On average low persistent income participants contribute 2.4% of their income to the TDA, while high persistent income participants contribute 6.0%. Contribution rates are much higher in period 2 for all income levels compared to period 1. They are close to the contribution limit of 10%.

No matching	TDA participation rate by income (for eligible employees)			TDA contribution rate by income (for TDA participants)		
	low	middle	high	low	middle	high
Period 1	28.9%	48.8%	86.4%	2.4%	4.3%	6.0%
Period 2	99.9%	100%	100%	9.9%	9.9%	10.0%
Period 3	0	0	0	0	0	0

[19] See Andrews (1992), Bassett et al. (1998), Munnell et al. (2002), and Huberman et al. (2007).

Table 3: Simulation results (without employer matching)

It is not surprising that income matters for 401(k) plan participation and contributions. This is because: (1) low-income employees face lower tax rates and benefit less from the tax-deferred nature of 401(k) plans than high-income employees; and (2) low-income employees face higher replacement rates from social security and therefore have less need for additional retirement income than high-income employees. Thus, the 401(k) plan participation rate and the contribution rate of low-income employees are smaller than those of high-income employees.

The period in the model, which represents the age of employees, is also related to both 401(k) plan participation rate and contribution rate. Different periods in the model reflect employees' different stages in the life cycle and their interests in retirement saving. Employees in period 2 are closer to retirement and normally their income is higher in period 2 than in period 1. Employees have stronger incentive to save in the second period. Thus, the participation rate and the contribution rate are higher in period 2 than in period 1.

4.2 Effects of Employer Matching

Here we consider a typical employer matching program: \$0.50 on the first dollar contributed by employees up to 6% of employment income. Table 4 presents the results.

Compared to the benchmark, the presence of employer matching dramatically increases 401(k) participation rates of young employees. For example, the participation rate of low persistent income young employees increases from 28.9% to 61.7%. The participation rate of high persistent income young employees increases from 86.4% to 100%. Higher participation rate under employer matching is because, all else being equal, offering matching increases the attractiveness of saving in 401(k) plans. We notice that the effect is strongest for low-income young employees.[20] However, employer matching has little effect on the participation of old employees since almost all eligible old employees choose to participate with or without employer matching.

Match 50% of first 6%	TDA participation rate by income (for eligible employees)			TDA contribution rate by income (for TDA participants)		
	low	middle	high	low	middle	high
Period 1	61.7%	85.8%	100%	4.6%	5.3%	5.8%
Period 2	100%	100%	100%	9.4%	9.8%	9.8%
Period 3	0	0	0	0	0	0

Table 4: Simulation results (with employer matching)

A surprising finding is that the average contribution rates of participants are lower in period 2 for all income levels when there is a 50% match of the first 6% of contributions compared to the benchmark when there is no employer matching. The average contribution rate of high persistent income employees also drops in period 1. This seems counterintuitive. We may expect that the presence of employer matching would increase both the participation rate and the contribution rate, because it produces a large initial return on employee contributions which supplements the advantage of tax deferral. Two explanations may account for this result. First, employer matching has both an income effect and a substitution effect. Whether the matching encourages employees' own contributions or not depends on which effect dominates. If the income effect dominates, employee contributions are likely to drop. Another influence comes from the broadened participation in

[20] This offers an immediate suggestion for a policy that can broaden participation for young employees in self-directed savings plans such as IRAs: the government could match the savers' contributions.

401(k) plans. The participation rate increases for young employees when there is employer matching. Many employees that previously would not participate in 401(k) plans will do so with employer matching. Contribution rates of these new participants are likely lower than those of employees that participate when there is no employer matching. Thus, the average contribution rate is lower due to a dilution effect.

Our model allows us to distinguish between these two explanations.[21] Next we control for the dilution effect and only focus on employees that participate in 401(k) plans under both situations (with and without employer matching). This experiment will give us a more precise view of the effect of employer matching on employee contributions. Table 5 summarizes the results.

	No matching			**Match 50% of first 6%**		
	low	middle	high	low	middle	high
Period 1	2.4%	4.3%	6.0%	6.2%	6.3%	6.3%
Period 2	9.9%	9.9%	10.0%	9.4%	9.8%	9.8%
Period 3	0	0	0	0	0	0

Table 5: Contribution rates after controlling for dilution effect

In period 1 (when employees are young), the average contribution rates for all income levels increase when there is employer matching, after controlling for the dilution effect. For example, low-income employees contribute 6.2% of their income on average to 401(k) plans when there is employer matching, while they only contribute 2.4% when there is no matching. These employees, therefore, try to take advantage of the matching because

[21] Unlike a regression based estimation framework, the advantage of the current model is that we can compare the contribution behavior of two groups of employees that are identical except employer matching.

employer matching provides a strong additional incentive for them to contribute. The substitution effect dominates the income effect for these employees.

In period 2 (when employees are old), the average contribution rates of employees that participate in 401(k) plans are lower for all income levels when there is employer matching. This is likely because employees choose to contribute more than the matching limit, 6%, in period 2 when there is no employer matching. Thus, employer matching only provides a limited incentive, and the income effect dominates the substitution effect in the second period when there is employer matching.

A number of previous studies have also examined the extent to which assets held in the tax-deferred accounts have "crowded out" other assets, or whether saving in tax-deferred accounts represents new saving for employees. The empirical evidence is mixed.[22] What is the effect of employer matching on retirement wealth in this model? Table 6 sheds some light on this issue.

Persistent income history	Consumption ratio in period 3
Low, Low	1.122
Middle, Middle	1.099
High, High	1.058

Table 6: Consumption ratio in period 3

In the table we track the persistent income history for three groups of employees that participate in the TDA with and without the typical employer matching. For employees in each group, we report the average of the ratio of consumption in period 3 when there

[22]For a discussion of whether saving in tax-deferred accounts represents new saving, see Bernheim (2002) and Gale and Orszag (2003) for excellent surveys.

is employer matching to that when there is no employer matching. Since employees consume all available resources in period 3, consumption level in period 3 can be regarded as retirement wealth. We find that employees that experience "low" persistent income in both period 1 and 2 consume 12.2% more under employer matching than the same employees with no matching. This means retirement wealth for these employees increases 12.2% when there is employer matching. Similarly, retirement wealth increases 9.9% and 5.8% under employer matching for employees that experience both "middle" and "high" persistent income, respectively.

This experiment suggests that employees for all income levels accumulate more retirement wealth under employer matching if they participate in the TDA.[23] Moreover, low-income employees enjoy a higher increase (in percentage terms) in retirement wealth than high-income employees. This may be because for high-income employees, part of employer matching is offset by a reduction in their own contributions. For low-income employees, their contribution rates are relatively low. These employees want to take advantage of the matching and do not reduce their own contributions that much. To summarize, the total contributions (employer and employee combined) are higher for employees that have employer matching than for those who have no matching contributions from employers.

4.3 Effects of Income Risk

Here we perform a number of experiments by changing the parameter values to examine the effects of income risk on 401(k) participation and contributions. Specifically, we consider higher persistent income shocks, higher transitory income shocks, a possible disastrous labor

[23]This reflects the following two factors: (i) the benefits from the tax-deferral feature, and (ii) employer matching increases total compensation in the experiment.

income draw, and a case where there is no income risk. We keep the typical employer matching program in each experiment.

4.3.1 Higher Persistent Income Shocks

Here we increase the variance of persistent income shocks, σ_ξ^2, from 0.02 to 0.04 and keep other parameters constant. This change makes the high persistent income even higher and the low persistent income lower in the model. Table 7 reports the results.

Match 50% of first 6%	TDA participation rate by income (for eligible employees)			TDA contribution rate by income (for TDA participants)		
	low	middle	high	low	middle	high
Period 1	56.3%	92.0%	100%	4.1%	5.2%	6.3%
Period 2	99.8%	99.8%	100%	9.2%	9.6%	9.9%
Period 3	0	0	0	0	0	0

Table 7: Simulation results (higher variance of persistent shocks)

Compared to Table 4, a higher variance of persistent income shocks lowers the participation rate and contribution rate for low persistent income employees in both periods. For example, the participation rate of low persistent income employees drops from 61.7% to 56.3% in period 1. Their average contribution rate also drops from 4.6% to 4.1%. This is because these employees have lower income now.[24] On the other hand, employee contribution rates of high persistent income employees increase in both periods because their income is higher now. All high persistent income employees still choose to participate in the TDA.

[24] Compared to Table 4, these employees have the same transitory income but lower persistent income.

4.3.2 Higher Transitory Income Shocks

Here we increase the variance of transitory income shocks, σ_u^2, from 0.02 to 0.04 and keep other parameters constant. This change makes the range of transitory income wider.[25] Table 8 shows the results.

Match 50% of first 6%	TDA participation rate by income (for eligible employees)			TDA contribution rate by income (for TDA participants)		
	low	middle	high	low	middle	high
Period 1	58.9%	80.5%	93.1%	5.2%	5.4%	6.1%
Period 2	99.8%	99.8%	100%	9.1%	9.5%	9.5%
Period 3	0	0	0	0	0	0

Table 8: Simulation results (higher variance of transitory shocks)

Compared to Table 4, a higher variance of transitory income shocks lowers the participation rates for all income levels in both periods except for high-income old employees. This is because some employees are worse off and hence they do not save. For the effect of a higher variance of transitory income shocks on employee contributions, we notice that the average employee contribution rates for all persistent income levels increase in period 1. This is likely due to two reasons: (1) employees that participate in the TDA want to save more since the risk is higher now; and (2) the participation rates are lower now. In period 2, however, average employee contribution rates are lower for all persistent income levels. This is likely because after all income risks are realized in period 2, employees find that they have saved more in period 1 (compared to that in Table 4) and thus reduce their contributions in period 2.

[25]We choose transitory income in the range from $-2\sigma_u$ to $2\sigma_u$.

4.3.3 Disastrous Labor Income Shocks

Carroll's (1997) version of the buffer-stock model of savings explicitly allows for the possibility of a disastrous labor income shock. In particular, labor income is modelled as being zero with some probability and following Equation (2) otherwise. Since then, many studies incorporate a disastrous labor income draw. For example, Cocco et al. (2005) consider a zero-income realization with probability 0.5%. Given that zero-income realization is too extreme in a three-period model, here we consider a 10% labor income realization with probability 0.5%.

Table 9 shows that a possible disastrous labor income draw has a large impact on the participation rate in period 1 when employees are young. For example, 82% of low persistent income employees choose to participate in period 1 when there is a possible disastrous labor income draw, while only 62% of them participate in Table 4. The average participation rate of middle persistent income employees also increases, while the participation rate of high persistent income employees drops. The drop of participation rate for high persistent income employees is because about 0.5% of them are hit by the disastrous income draw. This also happens in period 2 for all income levels.

Match 50% of first 6%	TDA participation rate by income (for eligible employees)			TDA contribution rate by income (for TDA participants)		
	low	middle	high	low	middle	high
Period 1	82.0%	94.8%	99.5%	4.4%	5.3%	6.1%
Period 2	99.5%	99.5%	99.5%	9.3%	9.7%	9.8%
Period 3	0	0	0	0	0	0

Table 9: Simulation results (possible disastrous labor income draw)

Compared to Table 4, a possible disastrous labor income draw has a small effect on contribution rates for employees that participate in the TDA. Note that the average contribution rate of low persistent income employees drops in period 1 when there is a possible disastrous labor income draw. However, given that the participation rate for low-income employees increases considerably, it is likely that employee contribution rates are higher for all income levels in period 1 when there is a possible disastrous labor income draw.

4.3.4 No Labor Income Shocks

In order to better understand the effects of income risk on 401(k) participation and contributions, here we consider a case where there is no income risk at all. Employees in the model draw their persistent income in period 1 and have the same persistent income in period 2.[26] Table 10 presents the results.

Match 50% of first 6%	TDA participation rate by income (for eligible employees)			TDA contribution rate by income (for TDA participants)		
	low	middle	high	low	middle	high
Period 1	100%	100%	100%	3.9%	4.5%	5.9%
Period 2	100%	100%	100%	10%	10%	10%
Period 3	0	0	0	0	0	0

Table 10: Simulation results (no income risk)

When there is no income risk, all eligible employees choose to participate in the TDA given their income levels in the model. In terms of employee contributions, the contribution rate increases with persistent income in period 1. Compared to Table 4, the employee

[26]Thus, we only have three persistent income levels since there is no transitory income shocks.

contribution rates of low- and middle-income employees drop in period 1, while the contribution rate of high-income employees increases a little bit. In period 2, the contribution rates for all income levels hit the limit, 10%. This experiment shows that income risk does play a big role in 401(k) participation and contributions.

4.4 Effects of a Higher Contribution Limit

The contribution limit is set at 10% in previous experiments. Here we consider a higher contribution limit, 15%, and examine its effect. Note that employer matching is also included in this experiment. Table 11 reports the results.

Match 50% of first 6%	TDA participation rate by income (for eligible employees)			TDA contribution rate by income (for TDA participants)		
	low	middle	high	low	middle	high
Period 1	61.8%	83.6%	95.2%	4.7%	4.9%	5.8%
Period 2	99.9%	99.9%	100%	12.5%	13.5%	13.6%
Period 3	0	0	0	0	0	0

Table 11: Simulation results (higher contribution limit)

Compared to Table 4, a higher contribution limit has a small effect on 401(k) participation rate. It lowers the participation rate for middle- and high-income young employees. For other employees, the participation rate does not change much.

A higher contribution limit also has a small effect on participants' contributions in period 1. For example, low persistent income employees contribute 4.6% in the first period when the contribution limit is 10%. They contribute 4.7% after the contribution limit increases to 15%. A higher contribution limit does have a large impact on contribution rates for old employees, however. With a higher contribution limit, the average contribution rate of low persistent income employees increases from 9.4% in Table 4 to 12.5% in period 2. The average contribution rate of high persistent income employees also increases from 9.8% to 13.6%.

The effect of the contribution limit on employee contributions varies with the periods. The reason is the following. When employees are young, their income is relatively low and they do not have strong incentives to save for retirement. An increase in the contribution limit is irrelevant for most employees because their contribution rates in this period are much lower than the contribution limit. In period 2, when employees are closer to retirement, they have higher income and stronger incentives to save. We notice that contribution rates in period 2 are very close to the contribution limit when the limit is 10%. This implies that many employees are restricted by the contribution limit. An increase in the contribution limit in the second period relaxes the restriction. Thus, employees will contribute more on average in period 2 if the contribution limit is increased.

5 Conclusion

Due to the increased popularity and rapid development of 401(k) plans in the United States, many empirical studies have examined the effects of employer matching in 401(k) plans. This paper also studies employer matching. We focus on the effects of income risk using a simple 3-period model. We find that a typical employer matching program increases the participation rate of young employees. Employer matching increases contribution rates of low-income and young employees, while it decreases contribution rates of high-income employees and old employees. Employees accumulate more retirement wealth under employer matching. We also find that income risk plays a big role in employees' 401(k) participation and contribution decisions. The effects differ for different types of income risk and vary with employee income and age.

Appendix: Numerical Solution

We use numerical dynamic programming techniques to approximate the decision rules as well as the value function. The dynamic program has three state variables in addition to period j: W_j^T, W_j^D, and z_j. We need to solve for three control variables in each period: the contribution rate, the withdrawal amount from the TDA, and consumption. We exploit the scale-independence of the maximization problem and rewrite the level variables as ratios to $\bar{Y}_1$ (where $\bar{Y}_1 = exp(\bar{y}_1)$). We use lowercase letters to denote them: $w_j^T = \frac{W_j^T}{\bar{Y}_1}$, $w_j^D = \frac{W_j^D}{\bar{Y}_1}$, $x_j = \frac{X_j}{\bar{Y}_1}$, $c_j = \frac{C_j}{\bar{Y}_1}$.

We discretize the state-space along the two continuous state variables, W_j^T and W_j^D. The model is solved using backward induction. We optimize using grid search.[27] In the last period ($j = 3$) the policy functions are trivial.[28] In previous periods, we calculate optimal decision rules for each possible combination of nodes, using stored information about the subsequent period's decision rules and value function. For points which do not lie on the state-space grids, we evaluate the value function using a bi-cubic spline interpolation along the two wealth dimensions. After computing the values of all the alternatives, we pick the maximum, thus obtaining the decision rules for the current period. This process is iterated until $j = 1$.

Once we determine the optimal decision rules for all possible nodes in each period, we conduct simulations. For each simulation, we first generate the income history of 9000 eligible employees (3000 for each persistent income state). We compute the TDA participation rate for employees and the average contribution rate for TDA participants in each period

[27] The grids are unequally spaced. They are finer for lower values of wealth.

[28] Employees withdraw all funds from the TDA at the beginning of last period and consume all available after-tax wealth.

by applying the optimal decision rules. Finally, we find the average of 100 simulations.

References

[1] Adda, J., Cooper R., 2003. Dynamic economics: Quantitative methods and applications. The MIT Press, Cambridge.

[2] Andrews, E.S., 1992. The growth and distribution of 401(k) plans. In: J. Turner and D. Beller (Eds), Trends in pensions 1992, p.149-176. Washington, D.C.: U.S. Department of Labor, Pension and Welfare Benefits Aadministration.

[3] Bailey, J., Nofsinger, J., O'Neill, M., 2003. A review of major influences on employee retirement investment decisions. Journal of Financial Services Research 23(2): 149-165.

[4] Bassett, W.F., Fleming, M.J., Rodrigues A.P., 1998. How workers use 401(k) plans: The participation, contribution, and withdrawal decisions. National Tax Journal 51(2): 263-289.

[5] Bernheim, D., 2002. Taxation and saving. In: A.J. Auerbach and M. Feldestein (Eds), Handbook of public economics, vol. 3: 1173-1249. Elsevier Science B.V.: Amsterdam, The Netherlands.

[6] Carroll, C., 1992. The buffer stock theory of saving: Some macroeconomic evidence. Brookings Papers on Economic Activity 2: 61-135.

[7] Carroll, C., 1997. Buffer-stock saving and the life-cycle/permanent income hypothesis. Quarterly Journal of Economics 112: 1-55.

[8] Carroll, C., Samwick, A., 1997. The nature of precautionary wealth. Journal of Monetary Economics 40: 41-71.

[9] Choi, J., Laibson, D., Madrian, B., 2004. Plan design and 401(k) savings outcomes. National Tax Journal 57(2): 275-298.

[10] Choi, J., Laibson, D., Madrian, B., Metrick, A., 2004. Optimal defaults and active decisions. Working Paper, Harvard University.

[11] Clark, R.L., Schieber, S.J., 1998. Factors affecting participation levels in 401(k) plans. In: O. Mitchell and S. Schieber (Eds), Living with defined contribution pensions: Remaking responsibility for retirement, p.69-97. Philadelphia, PA: University of pennsylvania Press.

[12] Cocco, J., Gomes, F., Maenhout, P., 2005. Consumption and portfolio choice over the life-cycle. Review of Financial Studies 18(2): 491-533.

[13] Employee Benefit Research Institute (EBRI), 1994. Salary reduction plans and individual saving for retirement. Issue Brief No. 155. Washington, D.C.: EBRI, November 1994.

[14] Engelhardt, G., Kumar, A., 2007. Employer matching and 401(k) saving: Evidence from the health and retirement study. Journal of Public Economics 91(10): 1920-1943.

[15] Even, W., Macpherson, D., 2005. The effects of employer matching in 401(k) plans. Industrial Relations 44(3): 525-549.

[16] Gale, W., Orszag, P., 2003. Private pensions: Issues and options. In: H. Aaron, J. Lindsay, and P. Nivola (Eds), Agenda for the nation. Washington, D.C.: Brookings institution.

[17] Gomes, F., Michaelides, A., 2005. Optimal life cycle asset allocation: Understanding the empirical evidence. Journal of Finance 60(2): 869-904.

[18] Gourinchas, P., Parker, J., 2002. Consumption over the life cycle. Econometrica 70(1): 47-89.

[19] Hubbard, G., Skinner, J., Zeldes, S., 1994. The importance of precautionary motives in explaining individual and aggregate saving. Carnegie-Rochester Conference Series on Public Policy 40: 59-125.

[20] Hubbard, G., Skinner, J., Zeldes, S., 1995. Precautionary saving and social insurance. Journal of Political Economy 103(2): 360-399.

[21] Huberman, G., Iyengar, S., Jiang, W., 2007. Defined contribution pension plan: Determinants of participation and contributions rates. Journal of Financial Services Research 31(1): 1-32.

[22] Huggett, M., Parra, J., 2006. How well does the US social insurance system provide social insurance? Working Paper, Georgetown University.

[23] Huggett, M., Ventura, G., 2000. Understanding why high income households save more than low income households. Journal of Monetary Economics 45(2): 361-397.

[24] Joulfaian, D., Richardson, D., 2001. Who takes advantage of tax-deferred saving programs? Evidence from federal income tax data. National Tax Journal 54(3): 669-688.

[25] Kusko, A., Poterba, J., Wilcox, D., 1998. Employee decisions with respect to 401(k) plans. In: O. Mitchell and S. Schieber (Eds), Living with defined contribution pen-

sions: Remaking responsibility for retirement, p.98-112. Philadelphia, PA: University of pennsylvania Press.

[26] Madrian, B.C., Shea, D., 2001. The power of suggestion: Inertia in 401(k) participation and savings behavior. Quarterly Journal of Economics 116(4): 1149-1187.

[27] Mitchell, O., Utkus, S., Yang, T., 2005. Turning workers into savers? Incentives, liquidity, and choice in 401(k) plan design. NBER Working Paper No. 11726.

[28] Munnell, A., Sundén, A., Taylor, C., 2002. What determines 401(k) participation and contributions? Social Security Bulletin 64(3): 64-75.

[29] Papke, L., 1995. Participation in and contributions to 401(k) pension plans: Evidence from plan data. Journal of Human Resources 30(2): 311-325.

[30] Papke, L., 2004a. Individual financial decisions in retirement saving plans: The role of participant-direction. Journal of Public Economics 88(1-2): 39-61.

[31] Papke, L., 2004b. Choice and other determinants of eEmployee contributions to defined contribution plans. Social Security Bulletin 65(2): 59-68.

[32] Papke, L., Poterba, J., 1995. Survey evidence on employer match rates and employee saving behavior in 401(k) plans. Economic Letters 49(3): 313-317.

[33] Storesletten, K., Telmer, C., Yaron, A., 2004. Consumption and risk sharing over life cycle. Journal of Monetary Economics 51(3): 609-633.

[34] US General Accounting Office, 1997. 401(k) pension plans: Loan provisions enhance participation, but may affect income security for some. GAO/HEHS-98-5, Washington, D.C., October.

[35] The Vanguard Center for Retirement Research, 2001. Automatic enrollment: Vanguard client experience. Valley Forge, PA: The Vanguard Group.

[36] The Vanguard Group, 2004. How America saves, 2004: A report on Vanguard defined contribution plans. www.vanguardretirementresearch.com.