

Division by and into Zero

Question:

How do you know that the answer to $5 \div 0$ is not zero?

Possible answers to the question are:

1. That can't be right. The answer must be zero because 5 divided by nothing must be nothing.
2. I used a calculator to do $5 \div 0$ and it read ERROR. This must be the calculator's way of saying the answer is not zero.
3. $5 \div 5$ is 1; $5 \div 1$ is 5; $5 \div .1$ is 50. It looks like the smaller the divisor, the larger the answer. Zero is very very very tiny. The answer to $5 \div 0$ must be very very very large.

Response 1 is not valid. It says zero is nothing. This is a too common misconception. Zero can refer to many things: a count of something, a position on a number line, an elevation, the place at which an atomic bomb explodes (ground zero), and so on. None of these things are "nothing".



Response 2 is reasoning based on a procedure, in this case, a procedure performed by a calculator. The response hints that there might be something wrong with the question of $5 \div 0$ itself. If you enter $5 \div 0$ on a calculator, some models display ERROR; other models display something else, BUT none of the models display '0'.

Response 3 provides a conceptual reason for why the answer is not zero. It also provides insight into what the answer might be.

What is the answer to $5 \div 0$?

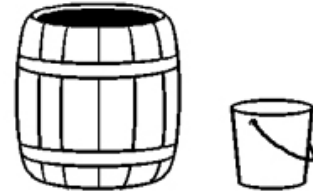
Before we answer the question, a quick revisit to the meaning of division is helpful.

Division means splitting up into equal groups. The goal is to reduce the stuff being split up so that zero stuff is left to split up. Two basic questions are normally asked when dividing:

-  How many groups are formed?
-  How many in each group?

We will use 'how many groups are formed' thinking to address the question, $5 \div 0 = ?$

Suppose the context for 'splitting up into equal groups' is using a bucket to empty the water out of a barrel.



We begin by considering the answer to $0 \div 5$.

How many groups are formed thinking:

$0 \div 5$ means that the barrel has zero litres of water in it and that the bucket holds 5 litres of water. The question becomes: How many buckets are needed to empty the barrel?

Because the barrel is already empty of water, you will need zero bucketfuls to empty it. Thus, it makes sense that $0 \div 5 = 0$.

What about the answer to $5 \div 0$?

How many groups are formed thinking:

$5 \div 0$ means that the barrel has 5 litres of water in it and that the bucket holds 0 litres of water. The question becomes: How many buckets are needed to empty the barrel?

Clearly, you are setting yourself up for a futile task. You will never be able to empty the barrel because your bucket holds 0 litres of water. You will be at the job **forever**. This is why the answer to $5 \div 0$ is sometimes referred to as 'infinity'.

Note: 'Infinity' is a concept, not a number.

You can also consider the question, $5 \div 0 = ?$ by doing arithmetic.

Consider the following arithmetic. If you are not sure of the answers, check them with a calculator.

- $5 \div 1 = 5$
- $5 \div .1 = 50$
- $5 \div .01 = 500$
- $5 \div .001 = 5\,000$
- $5 \div .0001 = 50\,000$
- $5 \div .00001 = 500\,000$
- $5 \div .000001 = 5\,000\,000$

You should notice that the answer is getting bigger and bigger (closer to infinity) as the divisor gets smaller and smaller (closer to zero). This leads to the conclusion that division by zero is not possible. This is why many calculators display ERROR when someone tries to divide by zero. It is the calculator's gentle way of saying, "you idiot, you can't do that arithmetic".

In conclusion, a good way to understand the answer to $5 \div 0$ is that the ANSWER IS NOT DEFINED. In other words, division by zero is not defined.

Refer to: [Dividing by Zero](#) if more help is needed.