Quantifying probability

Question:

How do you know that the probability of getting 1 HEAD & 1 TAIL when throwing 2 coins is 1/2?

Possible answers to the question are:

1. That can't be correct. Two coins can land showing 1 HEAD & 1 TAIL, 2 HEADS, or 2 TAILS. 1 HEAD & 1 TAIL is one of three possibilities. The probability must be 1/3, not 1/2.
2. I threw 2 coins 100 times to see what would happen. 1 HEAD & 1 TAIL came up 47 times out of 100 throws. 47/100 is pretty close to 1/2.
3. I imagined throwing a penny and a dime. I made a table of what can happen when you throw the two coins.

Penny	**Dime**
HEAD	HEAD
TAIL	TAIL
HEAD	TAIL
TAIL	HEAD

The table shows:

- 2 HEADS happening once
- 2 TAILS happening once
- 1 HEAD & 1 TAIL happening twice

There are four outcomes that can happen. Two of the four are 1 HEAD & 1 TAIL. The probability of getting 1 HEAD & 1 TAIL is 2/4 or 1/2.

Response 1 is not valid but it does indicate thinking about a model for what can happen. Unfortunately, the model ignores the reality that a HEAD on coin #1 and a TAIL on coin #2 is different from a TAIL on coin #1 and a HEAD on coin #2. Thinking in terms of a model concerns theoretical probability.

Response 2 is appropriate. It concerns experimental probability. This is the jargon used in the MB curriculum document. That jargon, however, is not appropriate. There really is no experiment. What is actually going on is that the probability situation can be enacted by throwing coins (in this case), by spinning a spinner, by pulling names out of a hat, by throwing dice, by counting how many car accidents involving males, by observing how successful a drug treatment is, etc. All of this involves collecting data about the probability situation and then using that data to obtain a probability value.

Experimental probability is determined by the fraction:

$$\frac{\text{number of wins}}{\text{number of tries}}$$

“wins” should be interpreted in a general way. It concerns whatever you are interested in (the desired outcome(s). For example, ‘wins’ could refer to the number of times a sum of 7 occurs when throwing two dice; the number of times a blue marble is pulled out of a bag containing marbles, the number of times a brake system fails when a car is driven, and so on. Tries (also referred to as trials) should be interpreted in a general way. It concerns, for example, the total number of ways two dice can land, the total number of times a marble was pulled out of a hat, the total number of times the brakes on the car were applied, and so on.

Response 3 is appropriate. It concerns theoretical probability because it involves a model of the situation. The model is indicated by listing ALL possible outcomes (HH, TT, HT, TH). The model assumes:

- A coin has an equal chance of landing on HEAD or on TAIL.
- The coins cannot remember what happened before.
- The coins are independent of each other (one coin does not influence what happens with the other coin).

Theoretical probability is determined by the fraction:

$$\frac{\text{Occurrences of desired outcome}}{\text{Total number of outcomes}}$$

In the example of the two coins, the desired outcome was 1 HEAD & 1 TAIL. That outcome occurred twice. The total number of outcomes was four. Thus theoretical probability of getting 1 HEAD & 1 TAIL is 2/4 or 1/2.

In the Manitoba grade 6 curriculum, only one event is to be considered (e.g. throwing one coin, throwing one die, picking one number out of a hat, etc.).

In grade 7 and beyond, at least two independent events can be considered. Examples of two independent events are: throwing two dice; picking two names out of a hat. Examples of three independent events are: throwing three coins; picking three names out of hat.

The term ‘sample space’ appears in the grade 7 curriculum. It is jargon for ‘all possible outcomes’.

For this reason, another way to write the fraction for theoretical probability is:

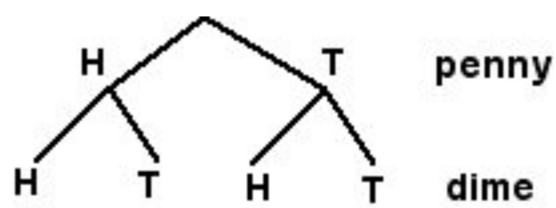
$$\frac{\text{Occurrences of desired outcome}}{\text{Sample space}}$$

A sample space can be indicated by two main ways: a list of all outcomes and a tree diagram.

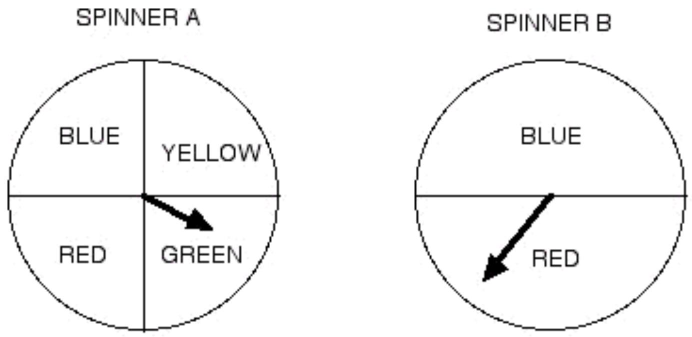
Let us revisit throwing a penny and a dime. The list way for indicating the sample space was already shown in a table format.

This is the list in a linear format: HH, TT, HT, TH.

A tree diagram for the penny and dime sample space is:

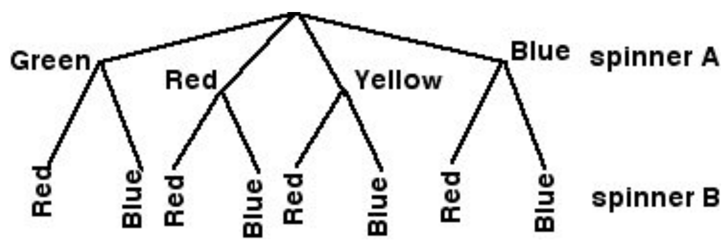


Let us consider another example. Suppose we spin two spinners. The diagram here shows the spinners as well as the list method (in table format) for indicating the sample space.



SPINNER A	SPINNER B
BLUE	BLUE
BLUE	RED
YELLOW	BLUE
YELLOW	RED
GREEN	BLUE
GREEN	RED
RED	BLUE
RED	RED

The tree diagram for the sample space is shown here. Follow the paths made by the branches and you will obtain the same outcomes as provided in the table (list method).



As an example of probability for the two spinners, the probability of both spinners stopping on RED is: 1/8.

To conclude, because the occurrences of the desired outcome(s) can range from zero to the same number as the sample space, the probability of something happening will have a value between 0 and 1. A probability of ‘0’ means that the outcome cannot happen. A probability of ‘1’ means that the outcome is guaranteed to happen. Probability values thus range from 0 to 1.

Refer to: Grade 6 Probability if more help needed.