Meanings of Fraction

Fractions are much more than pies

Fraction "fun" begins as early as Kindergarten where students have been known to say "*Cut the cookie in half and give me the bigger half.*" It continues as students encounter fraction notation, fraction equivalency, and operations with fractions. Why do fractions seem to be difficult for students?

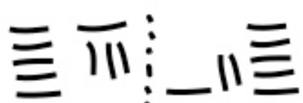
Our notation system can be a source of difficulty. When students encounter, for example, 2/3, they must understand that writing down two whole numbers does not represent a whole number. This can be magical from their point of view (Moss & Case, 1999).

Ancient cultures invented a variety of notation systems for fractions. The Mesopotamian and Chinese cultures did not invent special symbols for them. They extended their whole number system to name fractional amounts. Most ancient cultures invented a notation system for fractions that involved some form of fraction indicator. For example, the early Greeks placed an apostrophe-like mark above a counting number symbol. Some of the fraction notation systems were limited in the kinds of fractions they named. The early Egyptians, with a few exceptions, had symbols only for unit fractions (e.g. 1/5, 1/8). The Romans had symbols for a restricted set of fractions whose denominators were 12, 24, 48, and 72.

A glimpse of early fraction notation (Cajori, 1993) may help the reader appreciate students' struggles with fraction notation (see table 1 on next page).

This reading is an abridged version of an article in delta-K (Ameis, 2004)

Table 1. Ancient symbols for fractions

Ancient Culture	Culture's Symbol	Our Symbol
Egypt		1/5
Greece		1/9
Mesopotamia		11 32/100 (11.32)
Rome		7/12
India		2/3
China		58 125/1000 (58.125)

While a notation system can partly explain why students have difficulty with fractions, the variety of fraction meanings is another reason. The literature indicates that students are unlikely to resolve problem situations that involve fractions or understand fraction arithmetic unless they have a clear understanding of the full range of meanings of fractions (Tzur, 1999). Five meanings of fraction seem important for teaching middle years students.

Five meanings of fractions

The five meanings listed below serve as conceptual models or tools for thinking about and working with fractions and serve as a framework for designing teaching activities that engage students in sense making as they construct knowledge about fractions.

- Part of a whole
- Part of a group/set
- Measure (name for point on number line)
- Ratio
- Indicated division

For detailed discussion on the meanings, refer to: Five meanings of fraction

Note:

Another meaning of fraction is as an operator that enlarges or reduces something. This meaning is more abstract that the five listed above and does not have relevance in middle years curricula.

The meanings and working with students

How can we help students gain the power that ensues from understanding the listed five meanings of fractions and that allows them to move flexibly among the various meanings? How do we help them see fraction notation as something that makes personal sense? A partial response to these questions is implicit in the following description of my work with three grade 5 students on two Saturday mornings (part of a long-term study).

By the end of grade 4, the students had had some fraction instruction in school about the part of a whole meaning. They were proficient at naming pieces of pies/pizzas using fractions but they did not see 1/3 (for example) as a number distinct from a counting number. They considered a fraction to be two whole numbers that describe what happens when you cut up things like pies. They also had some difficulty understanding wholes and parts of wholes. For example when asked which is larger, 3 or 1/3, one of the students responded "*They are the same. If you used a whole pie there would be three thirds plus 3 on the bottom.*" As well, the students had little sense of why fractions were invented. In short, their knowledge of fractions was limited and confused.

I decided to redevelop fractions by first considering a reason for their invention. I used a measurement context for this, one that also made it possible to develop the name for a point meaning of a fraction. We imagined we were people from long ago that used a stick for measuring length. The students were asked to use the stick to measure the length of a table as accurately as possible. When asked how we could come up with a number for a part of a stick, the students suggested making equal marks on it and numbering them 1, 2, 3, etc. The measurement for the part of a stick would be the number closest to the actual length of the part. They were still thinking in terms of whole numbers even though they were subdividing the stick into smaller units of length.

When asked if there was another kind of number that could be used for naming a part of a stick, the students initially were unable to make use of their school-acquired fraction knowledge. The area model (pies/pizzas) that had been used for teaching fractions to them had not empowered them to carry fractions to other situations that might involve a different meaning. After we discussed sticks as objects that could be cut into pieces equal in length where each piece could be given a fraction name, the students realized that sticks were just like pies. They decided to split the stick into 8 pieces by making a succession of half marks on it and mentally attaching a fraction name to each mark (1/8, 2/8, etc.). We generalized measuring with a stick to constructing a number line (a ruler) that was then used to measure whole and fractional lengths.

We revisited the part of a whole meaning of a fraction, using it to attach fraction names in a variety of contexts (e. g. a loaf of bread). We discussed how we had made use of the part of a whole idea to make the marks on the ruler and how the fraction name for each mark was a different name for a point on the ruler. Labels for the two meanings were part of the discussion. The students now had a good sense of fraction as a part of whole and as a name for a point on a number line and an emerging understanding of the relationship between the two meanings. They realized that fractions were useful when measuring and when describing parts of things. They did not yet realize that people did arithmetic with fractions to solve problems. This would be the next step in the development of their fraction numeracy, a step that would utilize the name for a point meaning of fraction (measurement) as the vehicle.

References

Cajori, Florian. *A history of mathematical notations*. New York: Dover Publications, Inc., 1993.

Moss, Joan and Robbie Case. "Developing children's understanding of the rational numbers: A new model and an experimental curriculum." *Journal for Research in Mathematics Education* 30 (March 1999): 122-147.

Tzur, Ron. "An integrated study of children's construction of improper fractions and the teacher's role in promoting that learning." *Journal for Research in Mathematics Education* 30 (July 1999): 390-416.