Solving Equations

Question:

How do you know that $x = 3$ in the equation, $2x - 1 = 5$?

Possible answers to the question are:

1. If you use guess and check, the only number that works for x is 3.
2. Why should I care what x is?
3. You can think logically about it. If $2x - 1$ is 5, then 2x must equal 6 because $6 - 1$ is 5. If 2x is 6, this means x must be 3 because 2 times 3 is 6.

Response 1 concretes a valid approach but it only works well for simple equations. If the equation was '$(2x - 5)/3 + x/2 - 1 = 4x/3 + 2$', the guess and check strategy would be difficult to use and likely not lead to a successful solution.

Response 2 reveals a valid concern. Why should students learn about algebra in general, and in particular, about solving equations? There is no easy explanation. Real-world applications of algebra abound (for example, in launching a telecommunications satellite, in designing bridges, etc.). Unfortunately these applications are well hidden except to the experts who use them. Also, the applications require sophisticated understandings of science and engineering in order to make sense of the applications. To use an analogy, if someone cannot read, they would have little or no appreciation of what novels, magazine articles, and the like contain. It is sort of a catch-22 situation. If you are into science and engineering, then algebra makes sense and is appreciated. If you are not into science and engineering, then you would need to be "into" those matters, but in order to be "into" them you would need to appreciate and understand algebra.

Response 3 concerns a language-based strategy for determining the value of x. It is a basis of the symbolic method for solving equations. One would hope that students would be able to explain the symbolic method by using language-based arguments.

Note:

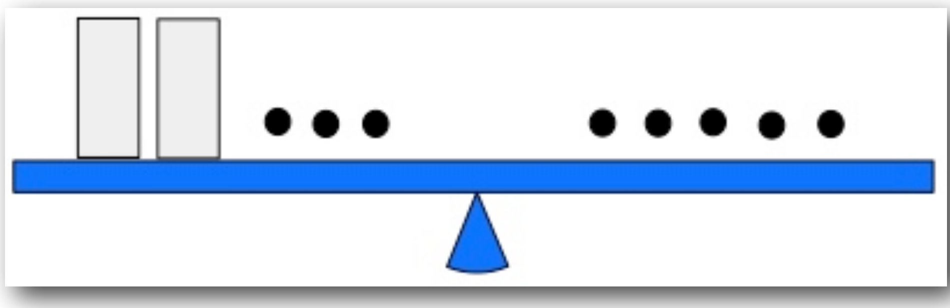
The matter of 'why learn algebra' is addressed after the discussion on teaching models for developing equation solving skills.

Models for teaching equation solving

The balance beam model and algebra tiles can be useful for developing equation solving skills. Each model has limitations. These are included in the discussion.

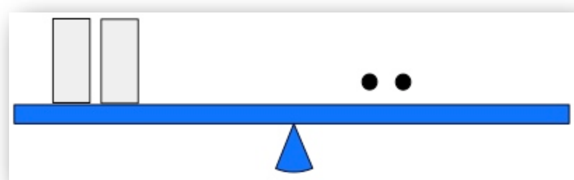
Balance Beam model

This model concerns the metaphor of a balance beam (teeter totter, in the eyes of children). It involves an analogy between equality of the two sides of the equation and a balanced balance beam. Whatever is done must maintain the balance of the beam. Here is an example for the equation: $2x + 3 = 5$.

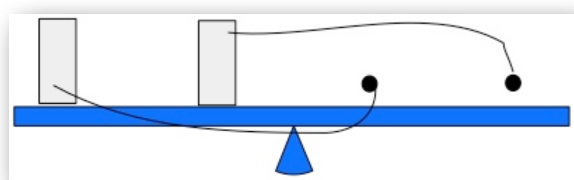


Each rectangle/box represents an 'x' (in the sense of we don't know how many dots are inside the rectangle/box) and each dot represents a count of 1. Thus, the Left Hand Side (LHS) shows $2x + 3$ and the Right Hand Side (RHS) shows 5. Solving the equation can proceed as follows.

We can remove 3 dots from the LHS. To maintain balance we must remove 3 dots from the RHS.



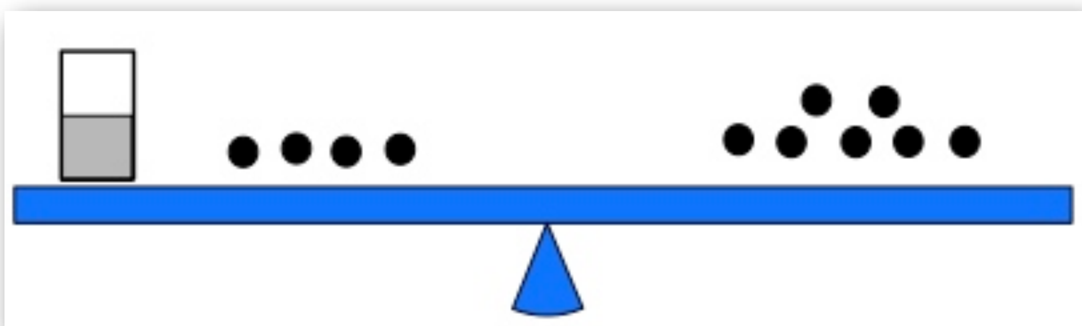
We can split the LHS into two equal parts and the RHS into two equal parts and then match each part. This tells us that the rectangle/box must have one dot in it to maintain balance. In other words, $x = 1$, is the solution for the equation.



There are three pedagogical limitations of the balance beam model:

- There are serious difficulties representing negative quantities. It is not feasible to represent such equations as 2x - 3 = 5 unless one resorts to complicated coding that flies in the face of common sense for children. After all, the balance beam model tends to be seen by them as a teeter totter and you can't put negative things on it.
- Physics may get in the way. For children, balance on a teeter totter depends on the weight of the people on it and also on where they sit. A balance beam is a lever and distance from the fulcrum (in this case, the distance from the centre pivot point) matters. Change the distance, and the balance will change, all other things remaining the same.
- It is tricky to model equations for which division of the variable is involved (e.g. x/2 + 5 = 8). One way to accomplish this is to use partially shaded rectangles/boxes. Here is an example for x/2 + 4 = 8.

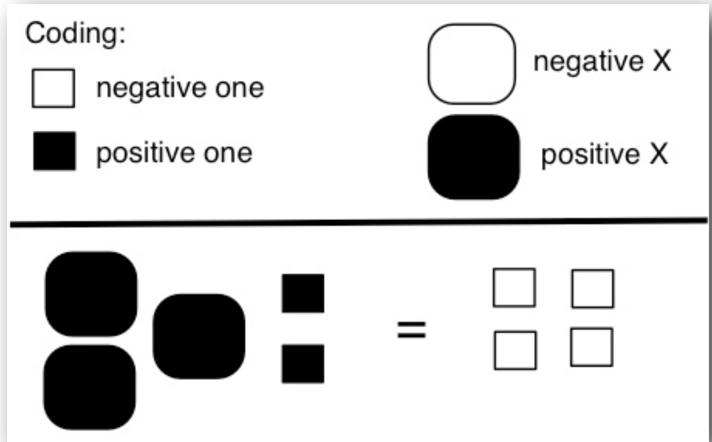
The rectangle/box that is half-shaded is intended to model x/2. This is not well accepted by enough students. Therefore, any subsequent processing is not going to work well if students do not make sense of the half-shaded rectangle/box representing x/2.



In conclusion, the balance beam model works best for equations that involve only positive quantities and whole number multipliers of the variable ('x'). The beam model is most useful for beginning the development of equation solving techniques for which the equations are restricted to positive quantities and whole number multipliers of 'x'.

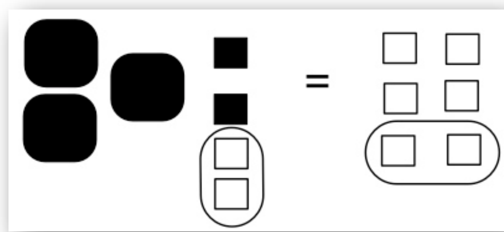
Algebra Tiles model

This model concerns using two types of counters (e.g a small and a large). The counters are, for example, colored red on one side and black on the flip side. The two colours can code positive and negative quantities. Here is an example for the equation: 3x + 2 = -4.



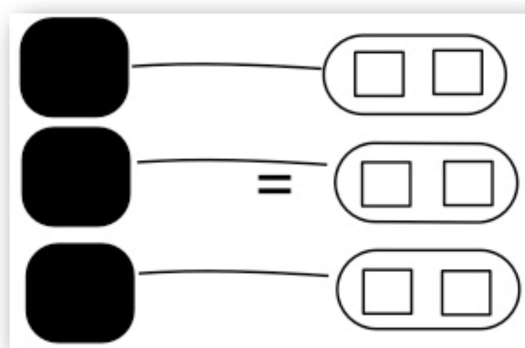
The processing for the algebra tiles model is similar to that for the balance beam model. It would look as follows for the equation, 3x + 2 = -4.

Add negative 2 to both sides.



On LHS, + 2 + (-2) = 0. In other words, the two positive and negative quantities cancel each other. On the RHS, you end up with 6 negative quantities.

Spit each side of the equal sign into 3 equal groups and match the groups. The diagram shows the solution, x = -2.



The algebra tiles model cannot model equations for which the variable is divided by a number (e.g. x/2 + 3 = 11). This model is best suited for equations involving integers and integer multipliers of the variable (e.g. -3x + 5 = -7).

Addressing the question of why learn algebra

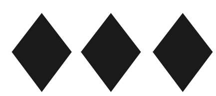
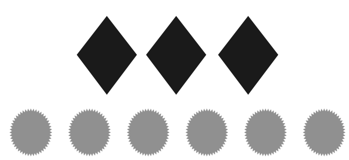
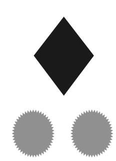
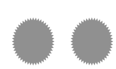
Two teaching strategies may help answer this question.

Using algebraic language in patterning activities. (Refer to Numerical Patterns)

The middle years Patterns & Relations strand concerns numerical patterning. At some point students are expected to use algebraic language to express a pattern. Suppose the input/output rule is double and add 3. The algebraic terminology for this is 2x + 3. Students should see that one use of algebra is as a short cut way of saying something in general. Hopefully, this will persuade some of them that algebra can be useful.

Using number tricks

Number tricks provide a way of both working with variables (and pseudo variables) and showing why variables (and thus algebra) are useful for generalizing work with numbers. Here is an example.

STEP	NUMERICAL EXAMPLE	PICTORIAL REPRESENTATION	ALGEBRAIC FORM
Choose a number	5		x
Multiply by 3	15		3x
Add 6	21		3x + 6
Divide by 3	7		x + 2
Subtract number first chosen	2		2

The numerical example for the number trick indicates that the result seems to be ‘2’. Picking other starting numbers would still result in ‘2’ at the end. However, numerical examples do not prove anything. All it takes is one counter-example and the trick is not true (in metaphorical speak, “one hole sinks the ship”). Numerical examples merely indicate what might be true.

The pictorial representation is a way of proving that the result will always be ‘2’. This involves understand the coding system and the processing steps. For the reader,

◆ represents the starting number (number chosen). It can be any number.

● represents a count of one. Thus ● ● represents 2 and so on.

The step, divide by 3, may be troublesome. What happens there is that the line above shows 3 ◆ and 6 ●. Dividing by 3 means making 3 equal groups. If you do that for 3 ◆ and 6 ●, you end up with ◆ and 2 ●.

The issue with the pictorial representation is that it is clumsy to use. Enter the algebraic form. It can be obtained from the pictorial form, by using the translations:

x is ◆ and 1 is ●

The number trick strategy can help students realize that algebra might have some use because it show show generalizing proves something an dhow using algebra language removes the clumsiness of working with pictures.

Refer to: Grade 8 Solving linear equations (8.PR.2) **if more help is needed.**