The Logic of Faith

St. Paul displays wit, together with a knowledge of classical Greek philosophy, in a remark to Titus about Cretan Christians:

> Even one of their own prophets has said, 'Cretans are always liars' Titus 1:12a.

Paul's reference is to Epimenides, a Cretan philosopher living in the 6th century BC. As a Cretan himself, Epimenides essentially says 'I am a liar.' We thus have the **Liar's Paradox**:

- If Epimenides **is** a liar then his statement is true, so that he is not a liar.
- On the other hand, if he is **not** a liar, then he makes a false statement, and is a liar!

If you prefer an orderly and logical world, you may feel an urge to dismiss this paradox as a mere semantic puzzle. This, however, would be a mistake; a reformulation of this paradox, known as Russell's paradox, is now regarded by philosophers and mathematicians as providing a fundamental insight into the nature of logical properties. Russell's criticism of Frege's formal logic helped to determine the modern form of *Set Theory*, the logical foundation of modern mathematics. Traces of the Liar's Paradox can also be seen in important mathematical results by Cantor, Turing and Gödel, which are built on the **diagonal argument**, a close logical cousin of the paradox.

Another staple of intro philosophy which is often too hastily answered is Anselm's **Ontological Proof** of the existence of God. For example, in *The God Delusion*, Dawkins dismisses Anselm's argument 'in so succinct and precise a manner as to make one pity theologians who have wasted their intellects on such questions.'[1] Anselm became Archbishop of Canterbury in the year 1093. His *Proslogion*, like Augustine's *Confessions*, is written in the form of a poetic and meditative prayer. In the midst of this prayer, Anselm thanks God for having revealed to him a logical demonstration of His existence. This demonstration is the Ontological Proof.

Proofs of the existence of God are controversial, and not just to atheists. On the one hand, St. Paul writes that

> what may be known about God is plain to (people), because God has made it plain to them. For since the creation of the world God's invisible qualities–his eternal power and divine nature–have been clearly seen, being understood from what has been made, so that (people) are without excuse.[2]

This would seem to assert that God and his attributes are knowable, even obvious. On the other hand, in another passage we read

[1] Reviewer Kevin Baldeosingh in *Trinidad and Tobago Newsday*
[2] Romans 1:19, 20.

'(S)ince in the wisdom of God the world through its wisdom did not know him, God was pleased through the foolishness of what was preached to save those who believe.'[3]

Here it is implied that discovering God is beyond the competence of human wisdom.

Concerning Anselm's argument, the famous atheist philosopher Bertrand Russell wrote in his *History of Western Philosophy* that 'it is easier to feel convinced that it must be fallacious than to find out precisely where the fallacy lies'. Like the Liar's Paradox, Anselm's proof – whether or not it proves what it claims to prove – raises interesting logical and mathematical issues. The following excerpt from a mathematical humour periodical[4] can be found in *Seven Years of Manifold (1968-1980)*:

THEOREM (due to Anselm, Aquinas, and others.) The Axiom of Choice is equivalent to the existence of a unique God.
PROOF:
$\Rightarrow$ (Assuming the equivalence of the Axiom of Choice and Zorn's Lemma.)
Partially order the set of subsets of the set of all properties of objects by inclusion. This set has maximal elements. God is by definition (according to Anselm) one of these maximal elements. Now

$$\text{God} \subset \text{God} \cup \{ \text{ existence } \}$$

hence

$$\text{God} = \text{God} \cup \{ \text{ existence } \}.$$

Therefore God exists.

To prove uniqueness, let God and God' be two gods. Then

$$\text{God} \cup \text{God'} \subset \text{God}$$

(according to Aquinas), therefore

$$\text{God'} \subset \text{God} .$$

Similarly God $\subset$ God'.
Hence God = God'.
$\Leftarrow$ Given a set $\{A_i : i \in I\}$ of sets, let the unique God pick $x_i \in A_i$ for each $i \in I$. (He can do so by omnipotence, proved as for existence above.) Then

$$(x_i)_{i \in I} \in \Pi_{i \in I} A_i$$

as required. **QED**

[3] 1 Corinthians 1:21.
[4] Yes, there are such things!

A layperson may have to accept on faith that some mathematicians find the above excerpt hilarious! I will attempt to explain the joke. First of all, pure mathematicians[5] do not generally expect the objects of their theories to have referents in the real world; for example, the infinitely fine lines of pure geometry are incapable of any physical realization[6]. A pure mathematician thus does not expect to theoretically prove the real existence of any being – let alone of the Ground of Being! The very subject of this proof therefore declares it to be crank, comic material. But here enters the tension which is a defining aspect of humour: the satire does identify legitimate analogies between Anselm's proof and standard techniques in mathematical Set Theory! The Axiom of Choice is a technically interesting assumption in Set Theory. Zorn's Lemma recasts this axiom in terms of the existence of maximal objects. Anselm characterizes God in the Proslogion as that than which nothing greater can be conceived. In mathematical terms, Anselm characterizes God as a maximal object. The various chains of reasoning in the proof follow standard mathematical patterns. The pastiche therefore connects with enough real mathematics, and with enough real theology, to have Pythonesque appeal.

Quoting a mathematical joke based on Anselm's proof is hardly a demonstration that criticism of the ontological argument is unfair. We can, however, see that 800 years before Cantor and Russell, Anselm was considering concepts which were later incorporated into Set Theory, which in turn is now universally accepted by mathematicians. We now turn to a sort of non-technical appeal to authority which suggests that Anselm's proof deserves at least a mathematical second glance. Engineers, physicists and others who have had some exposure to calculus will recognize the name of Gottfried Leibniz. Along with Sir Isaac Newton, Leibniz is regarded as co-inventor of calculus. The work of Leibniz in philosophy, in particular his *Theory of Monads*, is also still regarded as important. Students of the history of mathematics may be surprised to learn that Leibniz studied and accepted Anselm's proof! He in fact composed his own refinement of the proof, seeking to formalize and solidify Anselm's argument.

Leibniz worked in the 1600s, and nowadays, even his version of the calculus is known to have suffered from serious logical deficiencies. Modern logic only began in the nineteenth century with the work of Frege and others and then blossomed in the twentieth century with the deep and ground-breaking theorems of Kurt Gödel. Some argue that the three greatest logicians in history have been Aristotle, Frege and Gödel. Gödel proved mathematically the unsoundness of *Hilbert's program* and of *logical positivism.* His work revolved around a rigorous mathematical analysis of proof, provability and the logical dependence or independence of axioms. He spent the last part of his life at the Institute for Advanced Studies, and was Albert Einstein's closest friend for the last 15 years of Einstein's life. It is an indication of Gödel's genius that, having built his career in formal logic, he also made a major discovery in the Theory of Relativity. While modern mathematicians may dismiss some arguments made

[5]The distinction is between theoretical versus applied mathematicians.

[6]In fact, if the physical universe is grainy at a deep level, even as abstractions geometrical lines may not correctly model physical realities.

by Leibniz, few would call into question the logical and technical competence of Gödel.

Here then is the shocker: Late in his life, in the 1970s, Gödel circulated among his friends his own version of the Ontological Proof! His work in proof theory had led him to consider modal logic: the logic of possibility and necessity. It is just this flavour of logic which is necessary to analyze arguments such as Anselm's. Anselm begins by claiming that even the Fool must recognize God as a possible being – that is, as a being existing hypothetically, if not in actuality. He then presents a chain of reasoning leading to the necessity of Gods existence. Gödel carefully recast this argument in formal axiomatic logical terms.

My own intuitive response to Anselm's argument is skepticism, but I would certainly carefully consider any opinion of Gödel on questions of logic. I think that a mathematician, on learning of Gödel's endorsement, must hesitate to dismiss Anselm's argument as trivial, vacuous or devoid of logical content. Like the Liar's Paradox – a key ingredient in Gödel's famous theorems – the Ontological Proof may offer more than is first apparent.

A student of modern mathematical history will be aware of a balancing perspective on the above. At the time of his death, Gödel was suffering from acute paranoia. In fact, he literally starved himself to death because of a fear of being poisoned. This tragic circumstance deserves careful consideration.

All three"knights of the diagonal argument" – Cantor, Turing and Gödel – struggled with mental problems. Cantor spent much of his life in and out of sanatoria; Turing, a closeted homosexual, committed suicide; Gödel fought with mental health issues throughout his life. In fact, it was after Gödel's hospitalization in the 1940's for severe depression that Einstein gave him a course in the Theory of Relativity as a distraction. Gödel responded by finding new solutions to Einstein's equations, making a major new contribution to Relativity! Gödel's mental fragility, therefore, did not rule out high-quality insights.

Gödel's interest in Anselm's argument did not indicate merely a senile interest or paranoid obsession. Gödel, throughout his life, was deeply religious (as was Cantor.) Another of his lifelong interests was the work of Leibniz, his great predecessor. This was natural, given that Gödel was a philosophically inclined German-speaking mathematician. As we have seen, Leibniz had taken an interest in the logical structure of Anselm's proof, and thus Gödel came to Anselm's argument via Leibniz. As mentioned earlier, Gödel's technical tools meshed seamlessly with the issues discussd by Anselm. His interest in Anselm's proof was therefore organically related to long-standing themes and techniques of his thought.

We cannot rule out the possibility that Gödel's logical judgment had become impaired by the time of his work on the Ontological Proof. Nevertheless, that Anselm's argument could capture his attention at all, speaks to the enduring fascination of this logical construct of a medieval archbishop. Without endorsing or rejecting the Ontological Proof, I would suggest that Gödel's interest suggests that, like the Liar's Paradox, the Ontological Proof merits more than summary dismissal.