FORMULA FOR BASIC STATISTICS AND REGRESSION

Dec 2015

$$\bar{y}=\frac{\sum y}{n} \quad SS_y=\sum(y-\bar{y})^2=\sum y^2-\frac{(\sum y)^2}{n}=(n-1)s^2 \quad s_y^2=\frac{SS_y}{n-1} \quad s_y=\sqrt{\frac{SS_y}{n-1}} \tag{1}$$

$$H_0: \mu=\mu_0 \quad H_a: \begin{matrix}\mu\neq\mu_0\\ \mu>\mu_0\\ \mu<\mu_0\end{matrix} \quad t=\frac{\bar{y}-\mu_0}{\frac{s_y}{\sqrt{n}}} \quad df=n-1 \qquad F=\frac{n(\bar{y}-\mu_0)^2}{s^2_y} \quad df=1,n-1 \tag{2}$$

$$H_0: \mu_1-\mu_2=0 \quad H_a: \begin{matrix}\mu_1-\mu_2\neq 0\\ \mu_1-\mu_2>0\\ \mu_1-\mu_2<0\end{matrix} \quad t=\frac{(\bar{y}_1-\bar{y}_2)-0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}=\frac{(\bar{y}_1-\bar{y}_2)-0}{\sqrt{s_p^2(\frac{1}{n_1}+\frac{1}{n_2})}} \quad df=n_1+n_2-2 \tag{3}$$

$$s_p^2=\frac{SS_1+SS_2}{n_1+n_2-2} \qquad s^2_{\bar{y}}=\frac{\sum(\bar{y}_j-\bar{y}_G)^2}{k-1} \qquad F=\frac{n_j s_{\bar{y}}^2}{s_p^2} \qquad df=k-1, N-k \tag{4}$$

$$H_0: \mu_D=0 \quad H_a: \begin{matrix}\mu_D\neq 0\\ \mu_D>0\\ \mu_D<0\end{matrix} \quad t=\frac{\bar{D}-0}{\frac{s_D}{\sqrt{n_D}}} \quad df=\begin{matrix}n-1\\ n_D-1\end{matrix} \qquad s_D^2=s_1^2+s_2^2-2\times r_{12}\times s_1\times s_2 \tag{5}$$

$$SCP=SS_{xy}=\sum(x-\bar{x})(y-\bar{y})=\sum xy-\frac{(\sum x)(\sum y)}{n} \tag{6}$$

$$r=\frac{SCP}{\sqrt{SS_x SS_y}} \qquad \hat{y}=b_0+b_1x \qquad b_1=\frac{SCP}{SS_x} \qquad b_0=\bar{y}-b_1\bar{x} \tag{7}$$

$$SS_{Reg}=SS_{\hat{y}}=\sum(\hat{y}-\bar{y})^2=(n-1)s_{\hat{y}}^2=r^2SS_y \qquad r^2=\frac{SS_{Reg}}{SS_y} \tag{8}$$

$$SS_{Res}=SS_{y-\hat{y}}=\sum(y-\hat{y})^2=(n-1)s_{y-\hat{y}}^2=(1-r^2)SS_y \qquad (1-r^2)=\frac{SS_{Res}}{SS_y} \tag{9}$$

$$H_0: \rho=0 \quad H_a: \begin{matrix}\rho\neq 0\\ \rho>0\\ \rho<0\end{matrix} \quad t_r=\frac{r-0}{s_r}=\frac{r-0}{\sqrt{\frac{1-r^2}{(n-2)}}} \qquad df=n-2 \tag{10}$$

$$H_0: \beta_1=0 \quad H_a: \begin{matrix}\beta_1\neq 0\\ \beta_1>0\\ \beta_1<0\end{matrix} \quad t_{b_1}=\frac{b_1-0}{s_{b_1}}=\frac{b_1-0}{\sqrt{\frac{MS_{Res}}{SS_x}}}=t_r \quad df=n-2 \tag{11}$$

$$MS_{Res}=\frac{SS_{Res}}{n-2} \quad MS_{Reg}=\frac{SS_{Reg}}{1} \quad F=\frac{MS_{Reg}}{MS_{Res}}=\frac{\frac{r^2}{1}}{\frac{1-r^2}{n-2}}=t_r^2=t_{b_1}^2 \quad df=1,n-2 \tag{12}$$

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 \qquad b_1 = \frac{r_{y1} - r_{y2} r_{12}}{1 - r_{12}^2} \times \frac{s_y}{s_1} = b_{y1.2} \tag{13}$$

$$b_0 = \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2 \qquad R_{y.12}^2 = \frac{SS_{\hat{y}}}{SS_y} = r_{y\hat{y}}^2 \qquad 1 - R^2 = \frac{SS_{y-\hat{y}}}{SS_y} = r_{y(y-\hat{y})}^2 \tag{14}$$

$$SS_{Reg} = SS_{\hat{y}.12} = \sum (\hat{y} - \bar{y})^2 = (n-1) s_{\hat{y}}^2 = SS_y - SS_{Res} = R^2 \times SS_y \tag{15}$$

$$SS_{Res} = SS_{y-\hat{y}} = \sum (y - \hat{y})^2 = (n-1) s_{y-\hat{y}}^2 = SS_y - SS_{Reg} = (1 - R^2) \times SS_y \tag{16}$$

$$H_0: \rho_{y.12} = 0 \quad MS_{Reg} = \frac{SS_{Reg}}{p} \quad MS_{Res} = \frac{SS_{Res}}{n-p-1} \quad F = \frac{MS_{Reg}}{MS_{Res}} = \frac{\frac{R^2}{p}}{\frac{1-R^2}{n-p-1}} \quad df = p, n-p-1 \tag{17}$$

$$H_0: \beta_{y1.2} = 0 \quad H_a: \begin{matrix} \beta_{y1.2} \neq 0 \\ \beta_{y1.2} > 0 \\ \beta_{y1.2} < 0 \end{matrix} \quad t_{b_{y1.2}} = \frac{b_{y1.2} - 0}{s_{b_{y1.2}}} = \frac{b_{y1.2} - 0}{\sqrt{\frac{MS_{Res}}{SS_1 (1 - r_{12}^2)}}} \quad df = n-p-1 \tag{18}$$

$$SS_{Change} = SS_{\hat{y}1.2} = SS_{\hat{y}.12} - SS_{\hat{y}.2} = r_{y(1.2)}^2 \times SS_y \tag{19}$$

$$MS_{\hat{y}1.2} = \frac{SS_{\hat{y}1.2}}{1} \qquad F_{\hat{y}1.2} = \frac{MS_{\hat{y}1.2}}{MS_{Res}} = t_{b_{y1.2}}^2 \qquad df = 1,\ n-p-1 \tag{20}$$

$$r_{y(1.2)}^2 = \frac{SS_{\hat{y}1.2}}{SS_y} = R_{y.12}^2 - R_{y.2}^2 = r_{y(x1-\hat{x}1_{.2})}^2 \tag{21}$$

$$r_{y1.2}^2 = \frac{SS_{\hat{y}1.2}}{SS_y - SS_{\hat{y}.2}} = \frac{R_{y.12}^2 - R_{y2}^2}{1 - R_{y2}^2} = r_{(y-\hat{y}_{.2})(x1-\hat{x}1_{.2})}^2 \tag{22}$$

$$B_1 = \frac{r_{y1} - r_{y2} r_{12}}{1 - r_{12}^2} = b_1 \frac{s_1}{s_y} \qquad R_{Adj}^2 = 1 - (1 - R^2) \times \frac{n-1}{n-p-1} \tag{23}$$

$$SS_{\hat{y}1.23\cdots p} = SS_{\hat{y}.123\cdots p} - SS_{\hat{y}.23\cdots p} \qquad r_{y(1.23\cdots p)}^2 = \frac{SS_{\hat{y}1.23\cdots p}}{SS_y} = R_{y.123\cdots p}^2 - R_{y.23\cdots p}^2 \tag{24}$$

$$t_{y1.23\cdots p} = \frac{b_{y1.23\cdots p} - 0}{\sqrt{\frac{MS_{Residual}}{SS_1 (1 - R_{1.23\cdots p}^2)}}} \qquad Slope_{Nonlinear} = b_1 + 2 b_2 x \tag{25}$$

$$SS_{Total} = \sum_{j=1}^{k}\sum_{i=1}^{n_j} (y_{ij} - \overline{y}_G)^2 = (n-1)s_G^2 = SS_{Treatment} + SS_{Error} \qquad (26)$$

$$SS_{Error} = \sum_{j=1}^{k}\sum_{i=1}^{n_j} (y_{ij} - \overline{y}_j)^2 = \sum_{j=1}^{k} SS_j = \sum_{j=1}^{k} (n_j - 1)s_j^2 = SS_{Total} - SS_{Treatment} \qquad (27)$$

$$SS_{Treatment} = \sum_{j=1}^{k} n_j(\overline{y}_j - \overline{y}_G)^2 = n_j\sum_{j=1}^{k} (\overline{y}_j - \overline{y}_G)^2 = SS_{Total} - SS_{Error} \qquad (28)$$

$$\begin{array}{l} H_0: \mu_1 = \mu_2 = ... = \mu_k \\ H_a: Some = False \end{array} \quad MS_{Trt} = \frac{SS_{Trt}}{k-1} \quad MS_{Err} = \frac{SS_{Err}}{N-k} \quad F = \frac{MS_{Trt}}{MS_{Err}} \quad \eta^2 = \frac{SS_{Trt}}{SS_{Total}} \qquad (29)$$

$$\begin{array}{l} H_0: \mu_j = \mu_{j'} \\ H_a: \mu_j \neq \mu_{j'} \end{array} \quad t = \frac{\overline{y}_j - \overline{y}_{j'}}{\sqrt{MS_{Error}\left(\frac{1}{n_j} + \frac{1}{n_{j'}}\right)}} = \frac{q}{\sqrt{2}} \quad df = df_{Error} \qquad (30)$$

$$\begin{array}{l} H_0: \mu_j = \mu_{j'} \\ H_a: \mu_j \neq \mu_{j'} \end{array} \quad q = \frac{\overline{y}_j - \overline{y}_{j'}}{\sqrt{MS_{Err}\left(\frac{1}{n_j}\right)}} = \frac{\overline{y}_j - \overline{y}_{j'}}{\sqrt{\frac{MS_{Err}}{2}\left(\frac{1}{n_j} + \frac{1}{n_{j'}}\right)}} = t\sqrt{2} \quad df = df_{Error} \qquad (31)$$

$$\begin{array}{l} H_0: \mu_L = 0 \\ \quad\;\; \mu_L \neq 0 \\ H_a: \mu_L > 0 \\ \quad\;\; \mu_L < 0 \end{array} \quad L = \sum_{j=1}^{k} c_j\overline{y}_j \quad \sum c_j = 0 \quad SE_L = \sqrt{MS_{Err}\sum\frac{c_j^2}{n_j}} \quad t_L = \frac{L-0}{SE_L} \qquad (32)$$

$$SS_L = \frac{n_j L^2}{\sum c_j^2} \quad F_L = \frac{SS_L/1}{MS_{Err}} = t_L^2 \quad df_L = 1, df_{Err} \quad \eta_L^2 = \frac{SS_L}{SS_{Total}} \qquad (33)$$

(continues)

$$SS_{Err} = \sum_{a=1}^{A} \sum_{b=1}^{B} SS_{ab} = \sum_{a=1}^{A} \sum_{b=1}^{B} \sum_{i=1}^{n_{ab}} (y_{abi} - \overline{y}_{ab})^2 = \sum_{a=1}^{A} \sum_{b=1}^{B} (n_{ab}-1) s_{ab}^2 = SS_{SwAB} \quad MS_{Err} = \frac{SS_{Err}}{N - A \times B} \tag{34}$$

$$SS_A = \sum_{a=1}^{A} n_a (\overline{y}_a - \overline{y}_G)^2 \quad MS_A = \frac{SS_A}{A-1} \quad F_A = \frac{MS_A}{MS_{Error}} \quad \eta_A^2 = \frac{SS_A}{SS_y} \tag{35}$$

$$SS_B = \sum_{b=1}^{B} n_b (\overline{y}_b - \overline{y}_G)^2 \quad MS_B = \frac{SS_B}{B-1} \quad F_B = \frac{MS_B}{MS_{Error}} \quad \eta_B^2 = \frac{SS_B}{SS_y} \tag{36}$$

$$SS_{A \times B} = SS_y - SS_A - SS_B - SS_{Error} \quad MS_{A \times B} = \frac{SS_{A \times B}}{(A-1)(B-1)} \quad F_{A \times B} = \frac{MS_{A \times B}}{MS_{Error}} \quad \eta_{A \times B}^2 = \frac{SS_{A \times B}}{SS_y} \tag{37}$$

$$\begin{aligned} SS_{A \times B} &= \sum_{a=1}^{A} \sum_{b=1}^{B} n_{ab} (\overline{y}_{ab} - \{\overline{y}_G + [\overline{y}_a - \overline{y}_G] + [\overline{y}_b - \overline{y}_G]\})^2 \\ &= \sum_{a=1}^{A} \sum_{b=1}^{B} n_{ab} (\{\overline{y}_{ab} - [\overline{y}_a - \overline{y}_G] - [\overline{y}_b - \overline{y}_G]\} - \overline{y}_G)^2 \end{aligned} \tag{38}$$

$$SS_{AwB1} = \sum_{a=1}^{A} n_{a1} (\overline{y}_{a1} - \overline{y}_{.1})^2 \quad df_{AwB1} = A-1 \quad \sum_{b=1}^{B} SS_{Awb} = SS_A + SS_{AxB} \tag{39}$$

$$SS_{Total} = SS_A + SS_S + SS_{Error} \qquad SS_S = n_s \sum_{s=1}^{S} (\overline{y}_S - \overline{y}_G)^2 \tag{40}$$

$$SS_{Error} = \sum_{a=1}^{A} \sum_{s=1}^{S} (y_{as} - \{\overline{y}_G + [\overline{y}_a - \overline{y}_G] + [\overline{y}_s - \overline{y}_G]\})^2 = SS_{AxS} \tag{41}$$

$$SS_{Total} = SS_A + SS_B + SS_{AxB} + SS_S + SS_{AxS} + SS_{BxS} + SS_{AxBxS} \quad df_{AxBxS} = (A-1)(B-1)(S-1) \tag{42}$$

$$SS_{Total} = SS_A + SS_B + SS_{AxB} + SS_{SwA} + SS_{(BxS)wA} \quad df_{SwA} = A(S-1) \quad df_{(BxS)wA} = A(B-1)(S-1) \tag{43}$$

$$H_0: \mu_G = 0 \quad MS_G = \frac{N \times (\overline{y}_G - 0)^2}{1} \quad MSE_G = \frac{\sum_{i=1}^{N} (y_i - \overline{y}_G)^2}{N-1} \quad F = \frac{MS_G}{MSE_G} \quad df = 1, N-1 \tag{44}$$