– Why Symmetries Relate Scattering/Decay Amplitudes

- Time Evolution: One solution of Schrödinger equation for time dependence of a state is $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

  where H = Hamiltonian

- Suppose there is an $SU(2)$ symmetry $\vec{J}$
  - + This could be spin, orbital angular momentum, isospin, etc
  - + Total $\vec{J}$ operator is J^2 w/eigenvalue $j(j+1)$
  - + 3rd component operator is J_3 w/eigenvalue m
  - + Because this is a symmetry, an eigenstate $|j,m\rangle$ can only evolve to itself, ie,

    $$e^{-iHt}|j,m\rangle = M_j|j,m\rangle$$

    where M_j = constant
  - + Also by the symmetry, M_j depends only on the quantum number j and <u>not</u> quantum number m.

    For example, if $\vec{J}$ represents angular momentum, the symmetry is rotations. Spherical/rotational symmetry means we can rotate around the x axis to change where the z-axis points. That changes the m quantum number but can't otherwise change the time evolution

- Transition amplitudes
  - + The probability that we start w/state $|\psi\rangle$ and measure state $|\phi\rangle$ at time t is

    $$P = |\langle\phi|e^{-iHt}|\psi\rangle|^2$$
  - + This means the <u>probability amplitude</u> is $\langle\phi|e^{-iHt}|\psi\rangle$

  - + In a case with the $SU(2)$ symmetry write each state as a superposition of $|j,m\rangle$ states

    $$|\psi\rangle = a_1|j_1,m_1\rangle + a_2|j_2,m_2\rangle + \dots \qquad |a_1|^2+|a_2|^2+\dots = 1$$

    $$|\phi\rangle = b_1|j_1,m_1\rangle + b_2|j_2,m_2\rangle + \dots \qquad |b_1|^2+|b_2|^2+\dots = 1$$

+ We know eigenstates w/ different eigenvalues are orthogonal, so the probability amplitude is

$$\langle \phi | e^{-iHt} | \psi \rangle = \left(b_1^* \langle j_1, m_1 | + b_2^* \langle j_2, m_2 | + \cdots \right)\left(a_1 \mathcal{M}_{j_1} | j_1, m_1 \rangle + a_2 \mathcal{M}_{j_2} | j_2, m_2 \rangle + \cdots \right)$$

$$= b_1^* a_1 \mathcal{M}_{j_1} + b_2^* a_2 \mathcal{M}_{j_2} + \cdots$$

(states are normalized)

- Our examples using isospin
  + We can consider combinations of pions and nucleons and their isospin states (total)

$$\pi^+ + p = |\tfrac{3}{2}, +\tfrac{3}{2}\rangle, \quad \pi^- + n = |\tfrac{3}{2}, -\tfrac{3}{2}\rangle,$$

$$\pi^- + p = \left(|\tfrac{3}{2}, -\tfrac{1}{2}\rangle - \sqrt{2}\,|\tfrac{1}{2}, -\tfrac{1}{2}\rangle\right)/\sqrt{3},$$

$$\pi^0 + n = \left(\sqrt{2}\,|\tfrac{3}{2}, -\tfrac{1}{2}\rangle + |\tfrac{1}{2}, -\tfrac{1}{2}\rangle\right)/\sqrt{3}$$

  + It is impossible to have $\pi^+ + p \rightarrow \pi^0 + n$ or $\pi^- + p$ or $\pi^- + n$ b/c the I_z eigenvalue changes (also charge is not conserved)

  + However, the amplitudes

$$\mathcal{M}(\pi^+ + p \rightarrow \pi^+ + p) = \langle \tfrac{3}{2}, +\tfrac{3}{2} | \mathcal{M}_{3/2} | \tfrac{3}{2}, +\tfrac{3}{2} \rangle = \mathcal{M}_{3/2}$$

and

$$\mathcal{M}(\pi^- + n \rightarrow \pi^- + n) = \langle \tfrac{3}{2}, -\tfrac{3}{2} | \mathcal{M}_{3/2} | \tfrac{3}{2}, -\tfrac{3}{2} \rangle = \mathcal{M}_{3/2}$$

are equal!

  + We can also find the other scattering amplitudes in a similar way. The same calculations work for decays or can work for angular momentum

  + There is a generalization for other symmetries like SU(3)