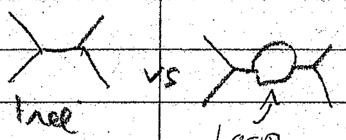
Feynman Diagrams

- General Feynman Diagrams & Rules

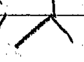
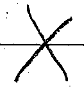
- To get decay rates + cross sections we need to calculate the amplitude $\mathcal{M}$
  + The calculation is technically in quantum field theory using perturbation theory (see Quantum Mech PHYS-4602)
  + Feynman diagrams are a graphic organization for these calculations w/ associated math (Feynman rules)

- Basics of Feynman Diagrams for scattering
  + External lines like —— for particles going into (initial) or out of (final) the diagram. Can be dashed, wavy, etc, to represent different types of particles
  + Internal lines look the same but stay in the diagram to represent virtual particles. Also called propagators
  + Vertices: lines joining together to represent particle interactions. While the rules for lines depend only on particle properties, vertices depend on the Hamiltonian.

  + Choose direction of time flow (left to right in Griffiths). Assemble all diagrams with the desired initial + final particles.
  + Assign 4-momentum to each line according to time flow. Conserve momentum at each vertex. Some



  + diagrams have loops with undetermined momenta — must integrate over these. We will look only at tree diagrams = no loops. (Griffiths describes differently)
  + Write each diagram in math using the Feynman rules. Add them to get $\mathcal{M}$

- Feynman Rules for Scalar/Pseudoscalar Particles
  - Vertices
    - ~~+~~Scalars typically have vertices with 3 or 4 lines with constant values, eg,  $= ig$ or  $= i\lambda$
    - + These can be oriented any way in a diagram
    - + Other "size" vertices are allowed but "non renormalizable". Same for other mathematical rules
  - External lines
    - + Each external scalar line gives a factor of 1
    - + How to justify this? If we FT in time: where p = 4-momentum of line, m = mass of particle, the

handwavy →

$$p^2 - m^2 = (p^0)^2 - (\vec{p}^2 + m^2) = 0 \to \left(-\frac{\partial^2}{\partial t^2} - (\vec{p}^2 + m^2)\right)\phi = 0$$

This is a harmonic oscillator $\phi \propto \exp(\pm i\sqrt{\vec{p}^2+m^2}\, t)$

- + The external line should give the amplitude of ϕ, like the

like the →

but we've already included the normalization $2E$ in phase space integral. So use 1

- + As an aside, FT in space also gives

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2 - m^2\right)\phi = 0 \longleftarrow = -(\partial_\mu \partial^\mu + m^2)\phi = 0$$

like "4D SHO"

This is the Klein-Gordon eqn

- Propagators / Internal Lines
  - + Each propagator has a factor —— $= i/(p^2 - m^2)$ where p = 4-momentum along the line, m = mass of particle
  - + NOTE: virtual particles do NOT have $p^2 = m^2$!
  - + Justification: The propagator is attached to a vertex at both ends. These modify the KG eqn to have forcing:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2 - m^2\right)\phi = e^{ip\cdot x}$$

← not the natural freq.

- + Like a forced oscillator, solution is

$$\phi = e^{ip\cdot x}/(p^2 - m^2)$$

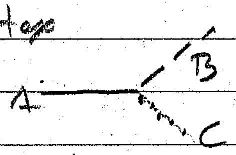
But notice that p^0 is not the natural frequency!

- + We take the amplitude $1/(p^2-m^2)$ and include a factor of i

- The ABC Theory
  - Suppose we have a model of 3 scalar particles A, B, C
    - Each has its own propagator

$$\underset{A}{\xrightarrow{p}} = \frac{i}{p^2 - m_A^2}, \quad \underset{B}{\overset{p}{\dashrightarrow}} = \frac{i}{p^2 - m_B^2}, \quad \underset{C}{\xrightarrow{p}} = \frac{i}{p^2 - m_C^2}$$

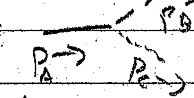
    - There is one vertex



$$= -ig$$

- Example: Decay of A if $m_A \geq m_B + m_C$
  - The only tree diagram is the vertex, so

$\mathcal{M} =$

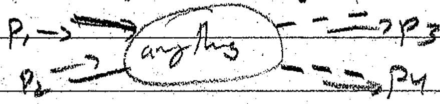


$= -ig$ ← Griffiths puts extra factor of i

  - Therefore, the decay rate is $\Gamma = \dfrac{g^2 p_f}{8\pi m_A^2}$

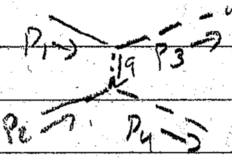
  If $m_B = m_C$, $p_f = \sqrt{m_A^2/4 - m_B^2}$ (see your homework)

- Example $A + A \to B + B$ scattering
  - We want all diagrams of the form

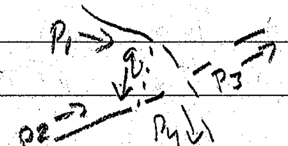


  - There are 2 diagrams

$\mathcal{M} =$



$+$



  You can "twist" the A lines or B lines, but not both.
  The choice is how they connect

  - Then

$$\mathcal{M} = (-ig)^2 \left(\frac{i}{(p_1 - p_3)^2 - m_C^2} + \frac{i}{(p_1 - p_4)^2 - m_C^2} \right) = \frac{-ig^2}{t - m_C^2} + \frac{-ig^2}{u - m_C^2}$$

  The diagrams are called "t-channel" and "u-channel"

  - The differential cross section is

Interference

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{128\pi^2 s} \frac{|\vec{p}_f|}{|\vec{p}_i|} \left(\frac{1}{t - m_C^2} + \frac{1}{u - m_C^2} \right)^2$$

$s = E_{cm}^2$ angle dependence

see HW

+ If $m_A = m_B \Rightarrow |\vec{p}_f| = |\vec{p}_i| = p$, $t = -2p^2(1-\cos\theta)$, $u = -2p^2(1+\cos\theta)$

~~If~~ also $m_C = 0$

$$\mathcal{M} = \frac{-ig^2}{2p^2}\left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}\right) = \frac{-ig^2}{p^2\sin^2\theta}$$

and

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{128\pi^2 s p^4 \sin^4\theta} \leftarrow \text{infinite range for } m_C = 0$$

- Other examples on homework!

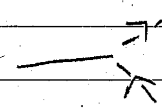
- Antiparticles: we've made a hidden assumption that the particles we're studying = their own antiparticles. What if they're different?

- Feynman Rules for lines
  + The particle lines (internal + external) have arrows that trace the particle number + can't reverse direction
  + External lines are

    $\xrightarrow{p}$ $\rightarrow$ = 1 particle in or out

    $\xrightarrow{p}$ $\leftarrow$ = 1 antiparticle in or out
  + Propagator

    $$\underset{p\rightarrow}{\longrightarrow} = i/(p^2-m^2)$$

    Take p in the direction of the arrow, or $-p$ in opposite
- Take the ABC theory with $C = \overline{B}$ = antiparticle of B
  + Then $m_C = m_B$
  + The vertex becomes  $= -ig$

    This gives $A \to B\overline{B}$ decay
  + There is no $A+A \to B+B$ scattering (violates conservation of B particle #), but $A+A \to B+\overline{B}$ exists.

    Diagrams differ in connection of p_1, p_2 to B or $\overline{B}$