Quantum Chromodynamics (QCD)

At low energies, QCD is very hard to calculate about hadrons. This is the high-energy theory.

① Feynman Rules

- Fermions / Quarks
  - Quarks/antiquarks have color/anticolor
    + We must account for color on quark lines
    + Color is really a vector b/c it's a quantum number. We can label these as c_A where

fundamental representation $\longrightarrow$ $c_R = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $c_B = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $c_G = \begin{bmatrix}0\\0\\1\end{bmatrix}$

    + These are also complete $\sum_A c_A c_A^\dagger = \mathbb{1}$

  - Quark lines
    + External lines must indicate the color

$\overset{p\rightarrow}{\longrightarrow\otimes}$ = incoming quark = $c_A u_s(p)$

$\overset{p\rightarrow}{\otimes\!\longrightarrow}$ = outgoing quark = $\bar{u}_s(p)\, c_A^\dagger$

$\overset{p\rightarrow}{\longleftarrow\otimes}$ = incoming antiquark = $c_A^\dagger \bar{v}_s(p)$

$\overset{p\rightarrow}{\otimes\!\longleftarrow}$ = outgoing antiquark = $c_A v_s(p)$

    + Color does not change along internal lines

$\otimes\overset{p\rightarrow}{\longrightarrow}\bullet = \dfrac{i(\not{p}+m)}{p^2-m^2}\,\mathbb{1}$ ← color identity matrix

- Gluons: massless vector bosons (like photons) that carry color + anticolor
  - Mathematically, the quark vectors c_A are in fundamental rep of $SU(3)$ group
    + We saw this before as (u,d,s) quarks in the 8-fold way
    + Combining quark w/ antiquark in 8-fold way gave an octet and a singlet

+ Gluons form an <u>octet</u> (aka <u>adjoint</u>) rep. of $SU(3)$ b/c color/anticolor is the same math.

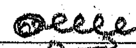
We represent these by $a^{\alpha}_{\phi} = \delta^{\alpha}_{\phi}$ where ϕ = color state $= 1\text{–}8$

+ Completeness $\sum_{\phi} a^{\alpha}_{\phi} (a^{\beta}_{\phi})^* = \delta^{\alpha\beta}$

• Gluon lines

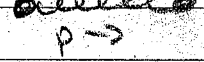
+ External lines represent the octet color state too

 = incoming gluon = $\epsilon^{(s)}_{\mu}(p)\, a^{\alpha}_{\phi}$

 = outgoing gluon = $\epsilon_{\mu}(p)^* (a^{\alpha}_{\phi})^*$

+ We <u>cannot</u> simply replace $\sum_{pol} \epsilon_{\mu}(p)\epsilon_{\nu}(p)^* \to -g_{\mu\nu}$ for gluons.

+ The color state does not change along the propagator

 $= \frac{-i g_{\mu\nu}}{p^2} \delta^{\alpha\beta}$

– Vertices

• Quark-gluon vertices must have a matrix that connects the octet state to color + anticolor

+ This is like how gamma matrices connect vector indices to spin states

+ These are the generators of $SU(3)$, they $= \lambda^{\alpha}/2$ where λ^{α} = <u>Gell-Mann</u> <u>matrices</u>

+ See Griffiths (8.34) for a listing

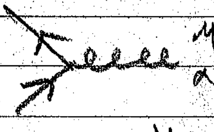
+ The Gell-Mann matrices have commutator

$$[\lambda^{\alpha}, \lambda^{\beta}] = 2i \sum_{\gamma} f^{\alpha\beta\gamma} \lambda^{\gamma}$$

for <u>structure</u> <u>constants</u> $f^{\alpha\beta\gamma}$. These are antisymmetric. Nonzero ones are in (8.36)

+ Useful identity: $Tr(\lambda^{\alpha}\lambda^{\beta}) = 2\delta^{\alpha\beta}$

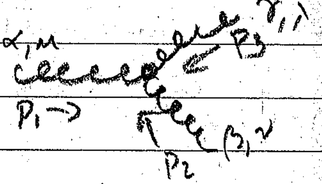
• Quark-gluon vertex is similar to QED



$= -i\frac{g}{2}\gamma^{\mu}\lambda^{\alpha}$ where g = coupling constant (similar to e)

Quark flavor doesn't change

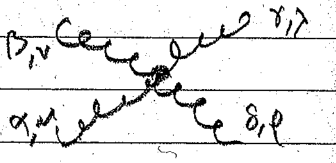
• Three-Gluon Vertex: Unlike photons, gluons interact w/ each other



$$= -g f^{\alpha\beta\gamma}\left[g_{\mu\nu}(p_1-p_2)_\lambda + g_{\nu\lambda}(p_2-p_3)_\mu + g_{\mu\lambda}(p_3-p_1)_\nu\right]$$

Note momenta point into the vertex. Reverse signs for any that point out

• Four Gluon Vertex

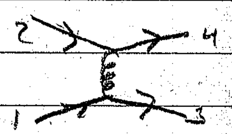


$$= -ig^2 \sum_\eta \left[f^{\alpha\beta\eta} f^{\gamma\delta\eta}(g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}) + f^{\alpha\delta\eta} f^{\beta\gamma\eta}(g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho}) + f^{\alpha\gamma\eta} f^{\delta\beta\eta}(g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho})\right]$$

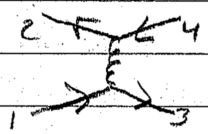
◎ Examples

– Quark/quark and quark/antiquark scattering

• If we consider different flavors, there is 1 diagram in each case



or



+ These have the same fermion+ propagator lines as $e^-\mu^-$ or $e^-\mu^+$ scattering. We know how to deal with those

+ What is new are the color factors

$$\frac{1}{4}\, c^\dagger(3)\lambda^\alpha c(1)\, c^\dagger(4)\lambda^\alpha c(2) \quad \text{or} \quad -\frac{1}{4}\, c^\dagger(3)\lambda^\alpha c(1)\, c^\dagger(2)\lambda^\alpha c(4)$$

• Nonrelativistic Scattering.

+ Nonrelativistically, the diagram for $e^-\mu^+$ scattering leads to Coulomb potential (translation of particle physics to EM)

$$V = -\frac{e^2}{r}$$

+ By analogy, quark/antiquark scattering must have nonrelativistic potential

$$V = -\frac{g^2}{r} f$$

for $f = \frac{1}{4} c_3^\dagger \lambda^\alpha c_1 \, c_2^\dagger \lambda^\alpha c_4$ = color factor

+ We can evaluate the color factor for a given quark/antiquark color state. The singlet is

$\frac{1}{\sqrt{3}}\left(|R\bar{R}\rangle + |B\bar{B}\rangle + |G\bar{G}\rangle\right)$

That is

$$f = \frac{1}{\sqrt{3}}\frac{1}{4} c_3^\dagger \lambda^\alpha \left(\begin{bmatrix}1\\0\\0\end{bmatrix}[1\,0\,0] + \begin{bmatrix}0\\1\\0\end{bmatrix}[0\,1\,0] + \begin{bmatrix}0\\0\\1\end{bmatrix}[0\,0\,1] \right) \lambda^\alpha c_4$$

$$= \frac{1}{4\sqrt{3}} c_3^\dagger \lambda^\alpha \lambda^\alpha c_4$$

But conservation of color implies c_3 + c_4 are also a singlet, so we do the same sum (by converting to a trace) + find $f = \frac{1}{12}\,\mathrm{tr}(\lambda^\alpha \lambda^\alpha) = 4/3$

+ So $V = -4g^2/3r$ = attractive. But can be repulsive for some color states

∘ Relativistic Scattering

+ This form of QCD doesn't really apply nonrelativistically. So let's consider quark/quark scattering from inside 2 protons that collide at LHC
+ We don't know the incoming quark colors, so average over them. And sum over outgoing colors.
+ So

$$\langle |M|^2 \rangle \propto \frac{1}{9}\left(\frac{1}{4}\right)^2 \sum_{\text{colors}} \left(c_3^\dagger \lambda^\alpha c_1 c_1^\dagger \lambda^\beta c_3\right)\left(c_4^\dagger \lambda^\alpha c_2 c_2^\dagger \lambda^\beta c_4\right)$$

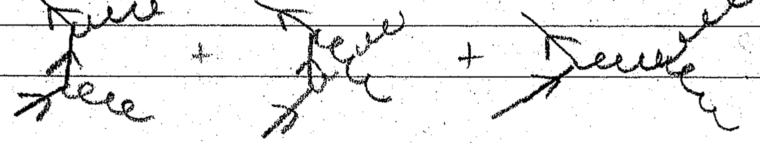
+ We can convert each factor into traces like we do for spinors

$$\sum_{\text{colors}} c_3^\dagger \lambda^\alpha c_1 c_1^\dagger \lambda^\beta c_3 = \sum_{\text{colors}} \mathrm{tr}\left[(c_3 c_3^\dagger)\lambda^\alpha (c_1 c_1^\dagger)\lambda^\beta\right]$$

$$= \mathrm{tr}\left[\lambda^\alpha \lambda^\beta\right] = 2\delta^{\alpha\beta}$$

– Quark/antiquark annihilation to gluons

∘ There are 3 diagrams



+ The first two are similar to $e^- + e^+ \rightarrow \gamma + \gamma$ annihilation in QED with additional color factors
+ The last diagram is a consequence of the 3-gluon vertex, which exists b/c gluons have color.
+ See the textbook for the (messy) formula)