PHYS-4303 Homework 9 Due 28 Nov 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/dcYrc2Yys2jsSrz` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Creation of Muons**

   Consider the process $e^+ + e^- \to \mu^+ + \mu^-$, ie, annihilation of an electron-positron pair to create a muon-antimuon pair. The electron mass is m and the muon mass is M, which is much larger than m.

   (a) Suppose the total CM frame energy is $2E$. Find the magnitude p_i of the initial $e^\pm$ momentum and the magnitude p_f of the final muons.

   (b) Draw the Feynman diagram for this process and write the probability amplitude in terms of spinors, gamma matrices, and metric tensors. Do not try to evaluate the amplitude.

2. **Gamma Matrix Identities**

   Prove each of the following gamma matrix identities. You may use the anticommutator

   $$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}1 \tag{1}$$

   and the fact that $\mathrm{Tr}(1) = 4$, where 1 is the 4×4 identity matrix that acts on spinors.

   (a) Show that $\bar{\gamma}^\mu \equiv \gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$. You may use the specific basis for gamma matrices given in the Griffiths text or our lecture notes to show whether γ^μ is Hermitian or anti-Hermitian for each value of μ.

   (b) Show that $\gamma_\mu\gamma^\mu = 4$ and $\gamma_\mu\gamma^\nu\gamma^\mu = -2\gamma^\nu$. These are the first two "contraction identities."

   (c) Show that $\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$ and $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho) = 4(g^{\mu\nu}g^{\lambda\rho} - g^{\mu\lambda}g^{\nu\rho} + g^{\mu\rho}g^{\nu\lambda})$. These are the first two "trace identities."