PHYS-4303 Homework 7 Due 14 Nov 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/dcYrc2Yys2jsSrz` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Average Lifetime** *from Griffiths 6.1*

    A particle decays with rate Γ.

    (a) What is the probability $P(t)dt$ that a particle decays between times t and $t + dt$ where dt is infinitesimally small? (This is the number of decays in that time range divided by the initial number of particles.)

    (b) Show that the average lifetime (ie, the average time when a particle decays) is $1/\Gamma$. *Hint:* remember the average of a quantity is its integral times the probability distribution.

2. **Decay Phase Space**

    In class, we saw that the decay rate for a particle with mass M to two product particles in the initial rest frame is
    $$\Gamma = \frac{Sp_f}{8\pi M^2}|\mathcal{M}|^2 \, , \tag{1}$$
    where p_f is the momentum of either final particle and $\mathcal{M}$ is the probability amplitude. The factor p_f comes from the phase space integral for the two final state particles.

    (a) What is the phase space factor p_f if both of the final particles has mass m? (This is the case, for example, in $K^0 \to \pi^+ + \pi^-$ decays.)

    (b) What is the phase space factor p_f if one of the final particles has mass m and the other is massles? (This is the case, for example, for decays $\pi^+ \to e^+ + \nu_e$ in the very good approximation that neutrinos are massless.)

    (c) For each case above, for what value of m is p_f maximized, and what is the maximum value? Give physical reasoning for your answer. (This is the case that gives the final particles the most phase space, ie, most possible final states, so it gives the highest decay rates typically.)

    (d) The decay cannot occur if the two final particle masses total to more than M (as we know from energy conservation). In each case above, how does p_f reflect that fact?