# PHYS-4303 Homework 5 Due 26 Oct 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/dcYrc2Yys2jsSrz` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Large Hadron Collider**

   The Large Hadron Collider (LHC) collides pairs of protons with a total energy of about 10 TeV ($= 10^4$ GeV) in their CM frame. For this problem, you will want to know that the mass of a proton is about 1 GeV.

   (a) What is the energy of one of the protons as measured in the rest frame of the other proton? *Hint:* If $p_1^\mu, p_2^\mu$ are the 2 initial 4-momenta, remember that gives the total CM frame energy and that $p_1 \cdot p_2$ gives the energy of the first proton in the 2nd proton's rest frame. Then relate the two products.

   (b) Suppose the LHC observes a collision that produces two photons and a number of other particles. The two photons have energies of 0.9 TeV and 0.6 TeV, and their paths are at an angle of 60 degrees to each other. If the collision process can be described as 2 protons become particle $X$ (plus other particles we don't care about), followed by particle $X$ decaying into the 2 photons, what is the mass of particle $X$?

2. **Mandelstam Variables** *Griffiths 3.25, 3.26, and many other sources*

   Consider a 2-to-2 scattering process $A + B \to C + D$ and define the three *Mandelstam variables*

   $$s \equiv (p_A + p_B)^2 \ , \quad t \equiv (p_A - p_C)^2 \ , \quad u \equiv (p_A - p_D)^2 \ . \tag{1}$$

   The masses of the particles are $m_A, m_B, m_C, m_D$ respectively.

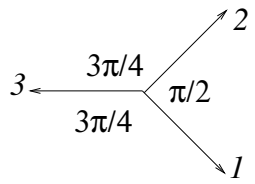
   (a) Prove that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

   (b) Show that the total CM frame energy is $\sqrt{s}$. *Hint:* this should be extremely short.

   (c) Suppose particles $A, B, C, D$ are all the same type of particle (like electrons), so the masses are all the same. Consider the collision in the CM frame. Then each particle has the same energy $E$ and (spatial) momentum of the same magnitude $p$. The particles scatter by angle $\theta$ (ie, particle "$C$" moves off at an angle $\theta$ compared to the initial velocity of "$A$"). Show that $t = -2p^2(1 - \cos\theta)$.

3. **3-Body Kaon Decay**

   A kaon of mass $M$ can decay into three pions, each of mass $m$. In this problem, a kaon decays at rest. Pions #1 and #2 move off perpendicularly to each other and at an angle of $3\pi/4$ to pion #3 (in a plane). See the figure below.




   Find the energy $E_1$ of pion #1. *Hint:* First show that pions #1 and #2 have the same energy using spatial momentum conservation. Then use 4-momentum conservation.