PHYS-4303 Homework 3 Due 3 Oct 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/dcYrc2Yys2jsSrz` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Rotations** *similar to Griffiths 4.21*

   For spin-1/2 states, the operator that carries out rotations of angle θ around an axis along unit vector $\hat{n}$ is

   $$U(\theta, \hat{n}) = \exp\left[i\theta\hat{n}\cdot\vec{\sigma}/2\right] \ , \tag{1}$$

   where $\vec{\sigma}$ are the Pauli matrices.

   (a) Write the operator corresponding to a rotation of 2π around the z axis for spin 1/2 as a 2×2 matrix. How does it compare to what you expected?

   (b) Construct the 2×2 matrix corresponding to a rotation of π around the x axis for spin 1/2. Show that it flips σ_z eigenstate with positive eigenvalue (spin "up") to the one with negative eigenvalue (spin "down"), up to an overall phase.

2. **Forbidden Decays** *inspired by Griffiths 4.37*

   The η meson is a fairly light, long-lived particle. You will want to look up its properties in the *Review of Particle Physics* particle listings (it is a light unflavored meson).

   (a) What are the isospin I, parity P, and charge conjugation C eigenvalus of the η?

   (b) Decays of η to 2 π mesons are not observed. What symmetry prevents this decay through strong or electromagnetic interactions? *Hint:* look up the discrete symmetries of the pions.

   (c) The decay $\eta \to \pi^0\gamma$ is also not observed. What symmetries prevent this decay from occuring through the strong force? What symmetries prevent this decay from occuring through the electromagnetic force? *Hint:* you will also need to look up symmetry eigenvalues for the photon, which appear only in the "particle listings" but not "pdgLive."

3. **Isospin and Decays** *inspired by Griffiths 4.32*

   The nucleons include the proton and neutron, and there are also excited states of nucleons known as N particles. For example, the $N(1710)$ is a nucleon with a mass near 1700 MeV which can be either positively charged (like an excited proton) or neutral (excited neutron). All nucleons have isospin $I = 1/2$; positively charged nucleons have $I_3 = +1/2$, and neutral nucleons have $I_3 = -1/2$. *Hint:* you will find the table of Clebsch-Gordon coefficients in the RPP to be helpful

   (a) By considering addition of isospin, estimate the ratio of the number of decays $N^+(1710) \to p + \pi^0$ to the decays $N^+(1710) \to n + \pi^+$. Remember that pions have $I = 0$ with I_3 equal to the pion charge.

   (b) Why do you think there may be more decays $N^+(1710) \to p + \eta$ than to either nucleon plus pion decay as discussed above? The η particle has $I = 0$.