PHYS-4303 Homework 2 Due 26 Sept 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/dcYrc2Yys2jsSrz` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Representations of U(1)**

   $U(1)$ is the group of 1×1 unitary matrices.

   (a) 1×1 matrices are (complex) numbers. Show that $U(1)$ matrices are phases $\exp(i\theta)$ where θ is real.

   (b) Show that the 2×2 matrices

   $$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

   satisfy the $U(1)$ multiplication rule, ie, they form a representation of $U(1)$.

2. **Adjoint Representation**

   (a) For any three matrices A, B, C, prove the *Jacobi identity*

   $$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0 \ . \tag{2}$$

   (b) Now take $A = T^a, B = T^b, C = T^c$ to be generators of some Lie group. Show that the Jacobi identity becomes

   $$\sum_d \left(f^{ade} f^{bcd} + f^{bde} f^{cad} + f^{cde} f^{abd} \right) = 0 \tag{3}$$

   in terms of the structure constants. *Hint:* Remember the commutation relation

   $$[T^a, T^b] = i \sum_c f^{abc} T^c \tag{4}$$

   (for any indices on the generators).

   (c) Define matrices t^a with elements $[t^a]^{bc} = -if^{abc}$. Then use your last result to show that these matrices satisfy (4), so they form a representation of the group algebra. This is called the *adjoint representation. Hint:* remember that the structure constants are totally antisymmetric.

3. **Parity, Charge, and Decays**

   For each listed particle decay below, indicate whether it can occur through strong or electromagnetic interactions or if it must proceed via weak interactions. Explain your reasoning. You will need to look up the parity eigenvalues and charge eigenvalues in the RPP; the particle listings show ($J^{P[C]}$) for spin and parity [and charge for C eigenstates]. (Some particles also show I^G, but not all. We'll define I later.)

   (a) $B_c^+ \to J/\psi + \pi^+$
   As mesons, B_c^+ is "bottom, charmed," J/ψ is a $c\bar{c}$ state, and pions are "light unflavored."

(b) *following Griffiths 4.37* $\eta \to 3\pi^0$
The η meson is also light unflavored.

4. **Parity and Spherical Harmonics**

For this question, look up the spherical harmonic functions either in Griffiths table 5.1 or the RPP.

(a) By writing the Cartesian components of position in terms of spherical polar coordinates, show that parity takes the spherical angle coordinates $(\theta, \phi$ to $(\pi - \theta, \phi + \pi)$.

(b) Show that the $l = 0, 1, 2$ spherical harmonics have parity eigenvalue $(-1)^l$. (This is in fact a general rule, so oribital angular momentum can complicate determining whether parity is conserved or not.)