PHYS-4303 Homework 10 Due 5 Dec 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/dcYrc2Yys2jsSrz` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **More on Muon Creation**

   This question builds on the first question from assignment 9. *Please see the solution set for that assignment. This is also filling in the steps for section 8.1 in Griffiths.* Recall the Mandelstam variables.

   (a) In the last assignment, you found the probability amplitude for muon creation via $e^+ + e^- \to \mu^+ + \mu^-$ to be

   $$\mathcal{M} = -\left(\frac{ie^2 g_{\mu\nu}}{(p_1+p_2)^2}\right)\bar{v}(2)\gamma^\mu u(1)\bar{u}(3)\gamma^\nu v(4) = -\left(\frac{ie^2}{(p_1+p_2)^2}\right)\bar{v}(2)\gamma^\mu u(1)\bar{u}(3)\gamma_\mu v(4)\ . \tag{1}$$

   Show that the square amplitude, when averaged over incoming spins and summed over outgoing spins, is

   $$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}\left(\frac{e^2}{s}\right)^2 \mathrm{Tr}\left[\gamma^\mu(\not{p}_1+m)\gamma^\nu(\not{p}_2-m)\right]\mathrm{Tr}\left[\gamma_\nu(\not{p}_3+M)\gamma_\mu(\not{p}_4-M)\right]\ . \tag{2}$$

   (b) Evaluate the traces and show that

   $$\langle|\mathcal{M}|^2\rangle = 8\left(\frac{e^2}{s}\right)^2\left[(p_1\cdot p_3)(p_2\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)+m^2(p_3\cdot p_4)+M^2(p_1\cdot p_2)+2m^2M^2\right]\ . \tag{3}$$

   (c) Work in the CM frame and let the electron and positron each have energy E. The angle between the incoming electron (e^-) and outgoing muon (μ^-) momenta is θ. Show that

   $$\langle|\mathcal{M}|^2\rangle = e^4\left\{1+\left(\frac{m}{E}\right)^2+\left(\frac{M}{E}\right)^2+\left[1-\left(\frac{m}{E}\right)^2\right]\left[1-\left(\frac{M}{E}\right)^2\right]\cos^2\theta\right\}\ . \tag{4}$$

   (d) Find the differential cross section and integrate over solid angle to find the total cross section

   $$\sigma = \frac{e^4}{48\pi E^2}\sqrt{\frac{1-M^2/E^2}{1-m^2/E^2}}\left[1+\frac{1}{2}\left(\frac{m}{E}\right)^2\right]\left[1+\frac{1}{2}\left(\frac{M}{E}\right)^2\right] \tag{5}$$

   and then take the limit for $E \gg m, M$. *Hint:* you found the initial and final momentum magnitudes on the last assignment.