# PHYS-4303 Homework 1 Due 19 Sept 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/dcYrc2Yys2jsSrz` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Fun and Profit with Natural Units**

   Recall that natural units are units with $\hbar = c = 1$. You may find the "Physical Constants" and "Astrophysical Constants" tables in the Review of Particle Physics useful.

   (a) Show that Newton's constant $G$ has units of (energy)$^{-2}$ in natural units (start with SI units and multiply by powers of $\hbar$ and $c$ to get a quantity with the right dimensions). Defining $G = 1/M_P^2$ in natural units, show that $M_P = 2.2 \times 10^{-8}$ kg.

   (b) The lifetime of a $\pi^0$ meson is about $8.4 \times 10^{-17}$ s. What is this in inverse MeV?

   (c) Hypothetical dark matter particles known as "WIMPs," if they exist, are expected to have cross sections of about $\sigma = 10^{-8}$ GeV$^{-2}$ for annihilating each other. (The cross section is the "target area" one particle has for another to hit it.) What is this cross section in SI units?

2. **Looking Things Up**

   The point of this problem is just to get you some familiarity with the RPP particle listings. For each part below, look up the requested quantity and report the value you find using eV units (or keV, MeV, etc) or a dimensionless number as appropriate to 2 significant digits. You may wish to use the interactive particle listings or the PDF summary tables, and you can use the "our fit" values in case of any conflict (since the RPP lists all up-to-date experimental results).

   (a) The mass of the $Z^0$ gauge boson.

   (b) The *width* of the $Z^0$ (a particle's width is its decay rate, which is the inverse of the lifetime).

   (c) The *branching fraction* of the $Z^0$ into hadrons. This is the fraction (or percentage) of $Z^0$ particles that decay producing hadrons and is listed under "decay modes" as "Fraction($\Gamma_i/\Gamma$)."

   (d) The mass difference between $K_L^0$ and $K_S^0$ mesons, listed with the data for $K_L^0$. $K$ mesons are "strange mesons," and you will need to convert to eV units.

3. **Pauli Matrices** *Essentially Griffiths 4.20*

   Consider the three Pauli spin matrices (as given in Griffiths equation (4.26) or your quantum mechanics textbook).

   (a) Show that the commutator is $[\sigma_i, \sigma_j] = 2i \sum_k \epsilon_{ijk}\sigma_k$, where $\epsilon_{ijk}$ is the Levi-Civita symbol.

   (b) Show that the *anticommutator* $\{\sigma_i, \sigma_j\} \equiv \sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij}\mathbb{I}$, where $\mathbb{I}$ is the identity matrix.

   (c) The Pauli matrices are the components of a vector of matrices $\vec{\sigma}$. Using the previous part, show that $(\vec{p}\cdot\vec{\sigma})^2 = \vec{p}^2\mathbb{I}$ for any vector $\vec{p}$.