# PHYS-3202 Homework 7 Due 8 Nov 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/H4t44ogzdTkskyD` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

**For this assignment, the following nomenclature is useful**. For orbits around the sun, the pericenter and apocenter are called *perihelion* and *aphelion* respectively (since *helion* refers to the sun in Greek). We also use *astronomical units* (abbreviated as AU) for lengths; 1 AU is equal to the semi-major axis of the earth's orbit around the sun. Likewise, we will use *years* to measure time; a year is the period of the earth's orbit. Finally, the symbol $\oplus$ denotes the earth, and $\odot$ denotes the sun.

1. **Changing Orbits** *from Fowles & Cassiday 6.19*

   A satellite orbits the earth (mass $M_\oplus$) with semi-major axis $a$. At some point in its orbit, it fires thrusters so its speed increases from $v$ to $v+\delta v$ (without changing the direction of motion). Find the change $\delta a$ of the semi-major axis, assuming that both $\delta v$ and $\delta a$ are small. *Hint:* you may work in a Taylor expansion for both $\delta v$ and $\delta a$.

2. **Hohmann Transfer** *adapted from Kibble & Berkshire and others*

   Suppose we want to send a space probe from earth to a planet farther from the sun. The most fuel-efficient orbit for the probe is known as a *Hohmann transfer*. Its perihelion is at earth's orbit, and its aphelion is at the other planet, which we will choose to be Jupiter. Assume that both earth and Jupiter have circular orbits (the eccentricities are less than 0.05 in both cases) with semi-major axes $a_\oplus$ and $a_J$ respectively.

   (a) Show that the perihelion distance (the minimum distance to the sun) of the probe's elliptical orbit is $r_{min} = a(1-e)$, where $a$ is the semi-major axis and $e$ the eccentricity. Similarly, show that the aphelion distance (maximum distance to the sun) is $r_{max} = a(1+e)$.

   (b) Find the required semi-major axis and eccentricity of the transfer orbit for the spaceprobe.

   (c) What is the transit time for the orbit? Give your answer first in terms of $a_\oplus, a_J$ and then in years given that $a_J \approx 5$ AU. Note that the transit time is half the period of the orbit.

3. **S2 and the Supermassive Black Hole**

   Genzel and Ghez split half the 2020 Nobel Prize in Physics for observations of the star S2 in the center of our galaxy. Star S2 orbits an object of mass $M$ called Sgr A$^*$ with a period of $T$ =16 years, semi-major axis of $a$ =1000 AU, and pericenter distance of 120 AU. Give your answers to the following questions to 1 significant digit. Assume that S2's orbit obeys Newton's law of gravitation.

   (a) What is the eccentricity $e$ of S2's orbit? *Hint:* you can use the formula for $r_{min}$ from the previous problem.

   (b) Find $M/M_\odot$, where $M_\odot$ is the mass of our sun. (You should find a large mass, which indicates that Sgr A$^*$ is a black hole.) *Hint:* use Kepler's third law. It is easier if you use AU and years for length and time units.

4. **Thin Plate**

Consider a thin plate of material, which is effectively two-dimensional (this is known as a *lamina*). In this problem, assume that the lamina lies in the $z = 0$ plane. Define $I_j$ as the moment of inertia around an axis passing through the origin parallel to the $j$ axis; that is, $I_x$ is around the $x$ axis, etc.

(a) *from Taylor* Prove that the moments of inertia $I_z = I_x + I_y$.This is sometimes called the *perpendicular axis theorem.*

(b) Assume that the lamina is a rectangle of sides with length $2a$ parallel to the $x$ axis and length $a$ parallel to the $y$ axis. The center of the rectangle is at the origin. The lamina has uniform mass surface density and total mass $M$. Calculate the moments of inertia of this lamina around the $x$, $y$, and $z$ axes (running through the origin). You may use the results of part (a) to simplify your calculations.