PHYS-3202 Homework 6 Due 1 Nov 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/H4t44ogzdTkskyD` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Angular Momentum of a Projectile**

   Consider a projectile of mass m launched from the origin with initial velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$, where y increases upward (y is the vertical coordinate). Calculate the angular momentum of the projectile as a function of time and show that its time derivative is equal to the torque due to gravity. You may assume that the projectile is near the surface of the earth (so the acceleration due to gravity is constant) and air resistance is negligible.

2. **Stable Circular Orbit**

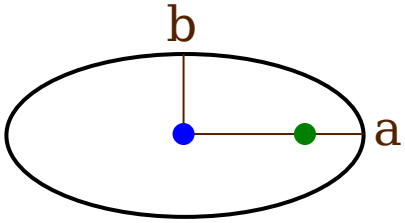
   A particle moves in a central potential $V = -mk/r$ with angular momentum per unit mass ℓ.

   (a) Use the effective potential to argue that a circular orbit with this angular momentum has radius $r = \ell^2/k$.

   (b) Show that the angular velocity $\dot{\theta}$ for the circular orbit is constant and find its value.

   (c) Suppose the radius is displaced very slightly from the circular orbit. Show that r oscillates around ℓ^2/k with frequency $\omega_0 = k^2/\ell^3$. *Hint:* Use a Taylor expansion to show that the conserved energy has an approximate form the same as a harmonic oscillator with that frequency.

3. Two particles of mass m move in elliptical orbits which are both identical to the ellipse in the figure below. The semi-major axis a and semi-minor axis b are shown as orthogonal brown lines and labeled at their endpoints with their lengths. Particle 1 is acted on by a central force directed toward the center of the ellipse, indicated by the blue dot in the figure. Particle 2 is acted on by a different central force directed toward the focus of the ellipse shown by the green dot in the figure. Both orbits have the same period T.



   What is the ratio of the speed v_1 of particle 1 at the point on the semi-minor axis labeled b in the figure to the speed v_2 of particle 2 at the same point in the figure? *Hint:* you do not need to know the specific forces; you just need to use Kepler's second law for the entire ellipse, the definition of angular momentum, and the fact that velocity is always tangent to the orbit.