PHYS-3202 Homework 5 Due 25 Oct 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/H4t44ogzdTkskyD` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Work Done on a Forced Oscillator** *similar to Cline 3.5*

   Consider a harmonic oscillator with damping α and natural frequency ω_0 that experiences a force $F(t) = F\cos(\omega t)$.

   (a) In class, we found the particular solution for x when the driving force is a complex exponential $F(t) = F\exp(i\omega t)$. If that complex solution is $x_1(t)$, show that $x(t) = (x_1(t)+x_1^*(t))/2$ is a solution for the cosine driving force in this problem. What is the particular solution $x(t)$ for the cosine driving force?

   (b) Remember that the work done on an object by a force F (in 1D) is the time integral of $F\dot{x}$. Find the work done on the oscillator by the force $F(t)$ over one period of length $2\pi/\omega$ for the particular solution (which is the late time solution neglecting transients). *Hint:* you will find angle addition formulas to be helpful when doing integrals.

   (c) The damping force is $-2m\alpha\dot{x}$. Find the power by the damping force at time t and the average power over one period for the steady-state solution. Show that your result is the opposite of the power from the driving force, so the total work done on the oscillator over a period is zero.

2. **Exponential Forcing**

   Consider a damped harmonic oscillator with natural frequency ω_0 and damping coefficient α. Suppose that it experiences a driving force $F(t) = F\exp(-\beta t)$ (with F, β real).

   (a) Show that the function $x(t) = A\exp(-\beta t)$ solves the equation of motion and find A (disregard any initial conditions). *Hint:* Newton's second law can be written as

   $$m\ddot{x} + 2m\alpha\dot{x} + m\omega_0^2 x = Fe^{-\beta t} \ . \tag{1}$$

   (b) Assuming $\alpha = 0$ (no damping), find the general solution and show that it oscillates with frequency ω_0 at late times. *Hint:* what are the solutions of the homogeneous differential equation?