PHYS-3202 Homework 3 Due 6 Oct 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/H4t44ogzdTkskyD` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Projectile Motion with Linear Drag**

   In the class notes, we showed that the position of a projectile that experiences gravity and linear drag is

   $$\vec{r}(t) = \frac{m\vec{g}}{\gamma}t + \frac{m}{\gamma}\left(\vec{v}_0 - \frac{m\vec{g}}{\gamma}\right)\left(1 - e^{-\gamma t/m}\right) \tag{1}$$

   as a function of time, where $\vec{v}_0$ is the initial velocity, $\vec{g}$ is the gravitational acceleration, and the drag force is $-\gamma\vec{v}$. The initial position is at the origin.

   (a) Show that this expression gives a parabolic trajectory in the limit of no air resistance $\gamma \to 0$. *Hint:* you will need to take a Taylor expansion of the exponential to evaluate the limit.

   (b) In general, it is not possible to find a closed form solution for the *range*, which is the x position where $z = 0$ at positive time. However, we can approximate it by the limit of x as $t \to \infty$. Find this limit in terms of the x component of the initial velocity.

2. **Circular Orbit**

   A satellite of mass m is in a circular orbit of radius r around a planet of mass M.

   (a) Find the total energy of this orbit.

   (b) The period of an orbit is the amount of time it takes for the satellite to return to its initial position (moving around a full circumference). Find the period.

   (c) Finally, suppose the planet rotates once in a time T. What is the radius of a circular orbit with the same period? This is known as a *synchronous orbit* since the satellite can stay over the same position on the planet.

3. **Multiple-Choice from a Previous Exam**

   For each part, choose all correct answer(s) from the options given and explain your answer in no more than two lines. There may be one or more correct answers for each part. (The variable k is a constant of appropriate units in any parts where it appears.)

   (a) A triathlete rides a bike up a hill. In what direction does the friction from the ground act on the bike? The wheels of the bike do not slip on the ground. *from MIT OpenCourseWare*
   A. $F_{friction} = 0$ B. down the hill C. up the hill D. perpendicular to the ground

   (b) For motion in the x direction only, which of the following forces is conservative? Here v denotes velocity, not speed.
   A. $F = -k$ B. $F = -kx$ C. $F = -kv$ D. $F = -kxv$

   (c) For motion in three dimensions, which of the following forces is conservative?
   A. $\vec{F} = ky\hat{\imath} - kx\hat{\jmath}$ B. $\vec{F} = kz^3\hat{k}$ C. $\vec{F} = ke^{kx}\hat{\jmath}$ D. $\vec{F} = ky\hat{\imath} + kx\hat{\jmath}$

   (d) $\vec{F}$ is a conservative force with potential energy $V(\vec{r})$. Which of the following give the work done by the force on an object moving from $\vec{r}_1$ to $\vec{r}_2$?

   A. $V(\vec{r}_1) - V(\vec{r}_2)$ B. $V(\vec{r}_2) - V(\vec{r}_1)$ C. $\int dt\vec{F}$ D. $\int_{\vec{r}_1}^{\vec{r}_2} d\vec{r} \cdot \vec{F}$