PHYS-3202 Homework 2 Due 27 Sept 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/H4t44ogzdTkskyD` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **A Particular Conservative Force** *Based on Idema 3.7*

   Consider the force $\vec{F} = (F_0/\ell^6)[(x^3y^3 + \alpha\ell^4 z^2)\hat{\imath} + \beta x^4 y^2 \hat{\jmath} + \gamma\ell^4 xz\hat{k}]$, where F_0 is a constant with units of force, ℓ is a constant with units of length, and α, β, and γ are dimensionless constants.

   (a) Find the values of α, β, and γ such that $\vec{F}$ is conservative. *Hint:* some of the constants may have an allowed range of values or be allowed if certain relationships are satisfied between them.

   (b) Suppose that α, β, and γ are chosen to make $\vec{F}$ a conservative force. What is the potential energy?

   (c) Suppose the particle starts at rest with a very small displacement from the origin. If the particle has mass m, what is its speed when it reaches the position $x = \ell$, $y = 0$, $z = \ell/2$? The force is the conservative force as given in the previous parts.

2. **Double Box Friction**

   A box of mass m rests on top of a box of mass M, which rests on a table. A rope is attached to the box of mass M.

   (a) What are the normal forces acting on the box of mass M?

   (b) With what magnitude tension must I pull the rope to cause the two boxes to start moving together? The coefficient of static friction between the lower box and the table is μ_1 and between the two boxes is μ_2.

   (c) Suppose both boxes are moving with the same velocity relative to the table, and I slowly increase the tension in the rope. At what magnitude of the tension does the upper box start to slip relative to the lower box? The coefficient of kinetic friction μ_k is the same between the lower box and either the table or the upper box.

3. **Turbulent Air Resistance**

   Consider an object falling in a uniform gravitational acceleration g against a quadratic drag force of magnitude ζv^2. In this problem, you will want to recall the hyperbolic trig functions and the relationships $\cosh^2\theta - \sinh^2\theta = 1$, $\tanh\theta = \sinh\theta/\cosh\theta$, $d\cosh\theta/d\theta = \sinh\theta$, and $d\sinh\theta/d\theta = \cosh\theta$.

   (a) Since the object falls straight down, write the velocity vector as $\vec{v} = -v(t)\hat{k}$ and find Newton's second law as a first order differential equation for $v(t)$.

   (b) Show that the speed of the object as a function of time is

   $$v(t) = \sqrt{\frac{mg}{\zeta}}\tanh\left(\sqrt{\frac{\zeta g}{m}}\,t\right) \ , \tag{1}$$

   where m is the object's mass. Assume that $v = 0$ at $t = 0$. Does this formula agree with the terminal velocity from the lecture notes? *Hint:* Integrate Newton's 2nd law by

separation of variables and a hyperbolic trig substitution $v = \sqrt{mg/\zeta}\,\tanh\theta$ (and use the identities above).

(c) Now find the distance traveled as a function of time. Check that your answer has the correct units.