PHYS-3202 Homework 12 NOT TO BE HANDED IN

This homework is solely for your use in studying and is not to be handed in.

1. **Inertia Tensor of Triangle**

   A thin lamina in the shape of an isosceles right triangle lies in the $z = 0$ plane with its right angle vertex at the origin and sides along the x and y axes. The edges of the triangle are at $y = 0$ for $0 \leq x \leq l$, $x = 0$ for $0 \leq y \leq l$, and $l = x + y$ between $(x, y) = (0, l)$ and $(l, 0)$. The lamina has a uniform surface density with total mass M. *Hint:* the perpendicular axis theorem for laminas on a previous homework will save some work.

   (a) Find the inertia tensor of the lamina in these coordinates. Use the symmetries of the object to reduce the number of calculations you have to do.

   (b) Find the unit vectors corresponding to the principal axes of the lamina and their principal moments by symmetry arguments, as follows: Start by noticing that the z axis and the line $x = y$ in the $z = 0$ plane must be principal axes by symmetry. Then choose the third axis perpendicular to those. Write the corresponding three unit vectors and check that they are eigenvectors by matrix multiplication, which gives the principal moments.

2. **Rotating Lamina**

   A rigid lamina (planar object) has principal moments I_1, I_2, and $I_3 = I_1 + I_2$ (this is the perpendicular axis theorem that we saw on a previous assignment). The components of the angular velocity along the corresponding principal axes are ω_1, ω_2, and ω_3 respectively. Show that $\omega_1^2 + \omega_2^2$ is constant. *Note:* you cannot assume that ω_3 is constant.