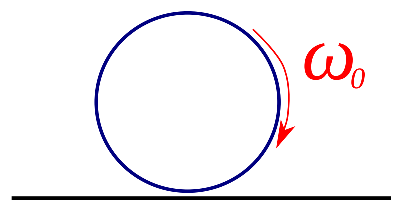
PHYS-3202 Homework 10 Due 29 Nov 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/H4t44ogzdTkskyD` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

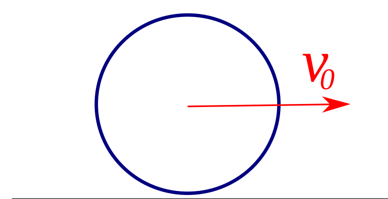
1. **Start Up of Rolling** *a problem seen many places*

   Consider a thin hoop of mass M and radius R that contacts a surface with coefficient of kinetic friction μ_k at time $t = 0$. It is oriented vertically, that is, with its axis of symmetry oriented horizontally, as in the figures below. The moment of inertia of the hoop is $I = MR^2$.

   (a) When the hoop contacts the surface, it has zero initial velocity and initial angular velocity ω_0, as in the figure below. When does the hoop stop slipping (ie, begin rolling without slipping)? How far does it travel before that time?



   (b) Suppose instead that the hoop has initial velocity v_0 in the $+x$ direction without rotating (initial angular velocity vanishes), as in the figure below. What is the speed of the hoop when it starts rolling without slipping?



2. **Pendulum in a Bus**

   A pendulum of length l hangs from the ceiling of a bus, which is waiting at a red stoplight (stationary with respect to the earth). At time $t = 0$, the light turns green, and the bus starts driving with constant horizontal acceleration $\vec{a}$. Describe the pendulum's position by θ, the angle from the downward vertical (parallel to the Earth's gravitational acceleration $\vec{g}$).

   (a) Find the equilibrium position θ_0 of the pendulum when the bus is accelerating. Describe how you find your answer from the perspective of both an inertial observer on the ground and an accelerating observer on the bus.

   (b) Describe the motion of the pendulum if it is hanging straight down at the time the bus starts accelerating. Assume $|\vec{a}| \ll g$. *Hint:* start by writing Newton's second law in the θ direction in the accelerating frame, then show it is approximately the same as for the harmonic oscillator.

3. **Rotating Cylindrical Space Vessel** *inspired by Kibble & Berkshire*

   One possible way residents of a space ship or space station can experience "artificial gravity" is if the vessel rotates around some axis. Consider a cylindrical vessel of radius R rotating with angular frequency ω around the cylinder axis.

   (a) The centrifugal force provides an effective gravitational force for stationary objects. At what height h above the "ground" at radius R a second story in a building have to be in order to have only 90% of the gravitational acceleration?

   (b) A train runs around the circumference of the vessel in the direction of the vessel's rotation with speed v relative to the ground. What is the increase in the effective weight (as measured by the normal force) for objects on the train compared to their weight at rest on the ground? Explain your answer first from the rotating frame of the vessel and then from the rotating frame in which the train is at rest. *Hint:* in the frame of the ground, the objects on the train have a centripetal acceleration.

4. **The Centrifugal Force is Conservative**

   A particle with position $\vec{r}$ relative to a reference frame rotating with angular velocity $\vec{\omega}$ experiences centrifugal force $\vec{F} = \vec{F} = m[(\vec{\omega}^2)\vec{r} - (\vec{\omega} \cdot \vec{r})\vec{\omega}]$. The reading by Tong claims that this force is conservative. To prove that, consider the potential

$$V = -\frac{m}{2}\left[\vec{\omega}^2\vec{r}^2 - (\vec{\omega} \cdot \vec{r})^2\right] \tag{1}$$

   and show that the gradient is $-\vec{F}$. Note that the gradient differentiates only the vector components and not the unit vectors because we are working in the rotating frame. *Hint:* Start by showing that $\vec{\nabla}(\vec{a} \cdot \vec{r}) = \vec{a}$ for any vector $\vec{a}$.