PHYS-3202 Homework 1 Due 20 Sept 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/H4t44ogzdTkskyD` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Vector Identity Applications**

   (a) Using vector triple-product identities, write $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ in terms of the dot products $\vec{a} \cdot \vec{c}$, $\vec{b} \cdot \vec{c}$, $\vec{a} \cdot \vec{d}$, and $\vec{b} \cdot \vec{d}$

   (b) *similar to problems by Taylor and by Thornton & Marion* An unknown vector $\vec{x}$ satisfies the following equations

   $$\vec{a} \times \vec{x} = \vec{b} \ , \quad \vec{a} \cdot \vec{x} = \phi \ , \tag{1}$$

   where $\vec{a}$ and $\vec{b}$ are known vectors and ϕ is a given scalar. Find $\vec{x}$ in terms of $\vec{a}$, $\vec{b}$, and ϕ. *Hint:* take the cross product of the first equation with a well-chosen vector.

2. **A Special Case of Circular Motion**

   Consider a particle that experiences an acceleration $\vec{a} = B\vec{v} \times \hat{k}$, where B is a constant and $\vec{v}$ is (as usual) the velocity. Assume that the particle moves entirely in the (x, y) plane.

   (a) Find $\ddot{x}$ and $\ddot{y}$ in terms of $\dot{x}$ and $\dot{y}$.

   (b) Show that circular motion $x = R\cos(\omega t), y = R\sin(\omega t)$ for some particular constant ω satisfies these equations and find ω.

3. **Elliptical Motion**

   Consider a particle moving in a plane with position $\vec{r} = a\cos(\omega t)\hat{\imath} + b\sin(\omega t)\hat{\jmath}$ with $a > b$.

   (a) Show that the x and y components of the position describe an ellipse, which has formula $x^2/a^2 + y^2/b^2 = 1$.

   (b) At what time(s) is the velocity $\vec{v}$ perpendicular to the position $\vec{r}$?

   (c) Find the squared speed v^2 and the times at which it is extremized. *Hint:* you do not need to find a derivative, although you may.