PHYS-4602 Course Project Instructions

This project is worth 15% of your grade for the entire course.

General Instructions

- The project is due at **10:59PM Mon 8 April 2024**.
- Submissions **must** be typed PDF files. They should be prepared with LaTeX or else MS Word (or similar word processor) with an equation editor for mathematics (*please export your Word file to PDF to submit*). Label your filenames with your first initial, last name, and "project" (for example `AFrey_project.pdf`); if you need to break your solution into multiple parts, label them in order with page numbers (`AFrey_project1.pdf`, `AFrey_project2.pdf`, etc). See the homework submission instructions on the course outline.
- Upload your submissions to `https://uwcloud.uwinnipeg.ca/s/FFJiJMNt9Czgo72` . **This is the same link as for homework.**
- You may use computer software (such as python, Maple, etc) to help solve any of the questions for the project, but you **must** attach your code (or worksheet, etc) as an appendix.
- There are 4 possible project choices below; you must confirm your choice of project with me via email by **1 April 2024**. Only one student may choose a given project, and the project will be assigned to the first student who chooses it. You may alternately design your own project but must consult with me on zoom to specify the project (by the same deadline). Confirming your project by the deadline is worth **10% of the project grade**.
- You will submit your project as a written report such as a lab report or essay, using full sentences. You do not need to include every step of mathematics, but you should explain what you are doing and where any equations come from. Each project report should have three sections: an introduction and one section for each of two approximation methods used to address the key problem of the project. Each section is worth **30% of the project grade**. The project descriptions below include the requirements for the specific sections.
- *Please make an appointment to discuss your project with me if you have questions or need help.* Note that, unlike homework assignments, I have not solved all these problems in advance, so it is appropriate to discuss how to work through difficult parts. Learning is more important than doing it all yourself, and some of these are hard problems.
- All references to the textbook are to Griffiths & Schroeter 3rd edition (abbreviated GS below). Let me know if you are using the 2nd edition.

Option 1: Yukawa Potential

This project is about the Yukawa potential described in GS problem 8.21 and 8.28. Use the notation of GS equation (8.73) for the Yukawa potential. The three sections should include the following:

1. **Introduction** Describe the Yukawa potential and its historical use — look it up and cite references (include a bibliography section at the end of your paper). Find the 3D Fourier

transform of the Yukawa potential and compare it to the formula for a propagator in particle physics. What does that suggest about the force carrier particle that creates a Yukawa potential?

2. **Variational Principle** Using the method suggested by problem 8.28, find the values of the photon mass μ where the variational principle guarantees that there is a negative energy bound state for an electron orbiting a proton.

3. **Perturbation Theory** Assuming $\mu a \ll 1$ where a is the Bohr radius, we can treat the Yukawa potential as the Coulomb potential plus a small perturbation. Estimate the ground state energy of the hydrogen atom if the photon has a small mass μ using first-order perturbation theory.

Option 2: Negative Hydrogen Ion

This project describes the ground state of the negative hydrogen ion as described in GS problems 8.8 and 8.25. You may cite results from the text if you can translate them appropriately. The three sections should include the following:

1. **Introduction** Describe the Hamiltonian for the negative hydrogen ion. Since the expectation value of the electron-electron repulsion potential from GS equation (8.17) will be important in both parts below, explain how to calculate it.

2. **Perturbation Theory** Treat the electron-electron repulsion potential as a perturbation of a Hamiltonian where each electron is separately feels a Coulomb attraction to the nucleus. Estimate the ground state energy to first order in perturbation theory.

3. **Variational Principle** Solve GS problem 8.25 to show that the ion does have a bound state.

Option 3: Van der Waals Interaction

GS problem 7.37 gives a 1D model of the van der Waals interaction between two atoms. We will extend this problem somewhat. The three sections should include the following:

1. **Introduction** Describe the model of GS problem 7.37 and how it models the van der Waals interaction. Solve part (a) of that problem.

2. **Perturbation Theory** Using your result from the introduction, solve part (d) of the problem to find the ground state energy to second order in perturbation theory. This result gives a potential energy for the separation R of the two heavy atomic nuclei.

3. **WKB Approximation** Add a kinetic energy to your result from the previous section to get a Hamiltonian for the relative motion of the two atomic nuclei in 1D. Use the WKB approximation to estimate the ground state wavefunction and energy. Assume that the ground state wavefunction is an even function in R. A symbolic algebra program should give the solution of the WKB integral in terms of a ${}_2F_1$ hypergeometric function (or we can try to work it out from properties of special functions).

Option 4: Computer vs Pencil

This option requires the use of computer software. In this project, you will compare a numerical method to estimate the ground state to an analytical one for a model potential. The three sections should include the following:

1. **Introduction** Discuss the advantages and disadvantages of numerical approaches to physics problems in comparison to analytic methods. Why is it important to test a numerical method? (You may want to write this section after you've worked out the next two.)

2. **Variational Method** Solve GS problem 8.30 parts (a) and (c). Repeat part (c) for the potential

$$V(x) = \begin{cases} \infty & x < 0 \\ V_0 x^2/a^2 & 0 \leq x \leq a \\ \infty & x > 0 \end{cases} .$$

3. **WKB Approximation** Use the WKB approximation to find the ground state wavefunction and energy for both the potentials considered above. How do your answers compare?

Option 5: Design Your Own Project

You will need to think about a project similar to the other 4 options and discuss it with me on zoom.