PHYS-4602 Homework 9 Due 20 March 2024

This homework is due to `https://uwcloud.uwinnipeg.ca/s/FFJiJMNt9Czgo72` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Matrix Perturbation Theory**

   Consider the matrix Hamiltonian
   $$H \simeq \begin{bmatrix} E_1 & \epsilon \\ \epsilon & E_2 \end{bmatrix} \tag{1}$$
   with $E_1 \neq E_2$ except when you are told otherwise. Assume that $\epsilon \ll E_1, E_2$.

   (a) To first order in perturbation theory, find the energy eigenvalues and eigenstates.

   (b) What is the first order correction to the energy if $E_1 = E_2 = E$?

   (c) Find the energy eigenvalues to second order in perturbation theory.

   (d) Find the energy eigenvalues and eigenstates exactly. Then expand them as a power series in ϵ and compare to your perturbative answers from parts (a,c). In the case that $E_1 = E_2 = E$, how does your answer compare to part (b)?

2. **Weak-Field Zeeman Effect**

   In the class notes, we stated that placing a hydrogen atom in a constant magnetic field $B_0\hat{z}$ introduces a contribution to the hydrogen atom of $H_1 = (e/2m)B_0(L_z + 2S_z)$. If this contribution is larger than the energy level splitting due to fine structure, this gives the "strong-field" Zeeman effect that we discussed in class. In this problem, consider the opposite limit, in which H_1 is smaller than the fine structure splitting. In this case, we include the fine structure corrections in the "unperturbed" Hamiltonian H_0 and treat H_1 as the perturbation to that.

   (a) With fine structure included, the eigenstates of H_0 are identified by n, total angular momentum quantum number j, its z component m_j, and the total orbital angular momentum quantum number ℓ (as well as total spin $s = 1/2$); the z-components m_ℓ and m_s are not good quantum numbers. Write $H_1 = (e/2m)B_0(J_z + S_z)$ since $\vec{J} = \vec{L} + \vec{S}$ and show that the change in energy due to B_0 is
   $$E^1_{n,j,m_j,\ell} = \frac{e\hbar}{2m} B_0 m_j \left[1 \pm \frac{1}{2\ell+1}\right] . \tag{2}$$
   To do this, you will need to know that the eigenstate of J^2, J_z, and L^2 is written
   $$\begin{aligned} |j = \ell \pm 1/2, m_j, \ell\rangle &= \sqrt{\frac{\ell \mp m_j + 1/2}{2\ell+1}} |\ell, m_\ell = m_j + 1/2, m_s = -1/2\rangle \\ &\quad \pm \sqrt{\frac{\ell \pm m_j + 1/2}{2\ell+1}} |\ell, m_\ell = m_j - 1/2, m_s = 1/2\rangle \end{aligned} \tag{3}$$
   in terms of the eigenstates of L^2, L_z, and S_z. *Hint:* It may be useful to note that $[H_0, J_z] = [H_1, J_z] = 0$.

(b) The quantity in square brackets in (2) is called the Landé g factor. Show that the g factor can also be written as

$$\left[1 + \frac{j(j+1) - \ell(\ell+1) + 3/4}{2j(j+1)}\right] , \tag{4}$$

which is the form given in Griffiths. You can start with (4) and try $j = \ell \pm 1/2$ separately to get the form given in (2).