PHYS-4602 Homework 6 Due 28 Feb 2024

This homework is due to `https://uwcloud.uwinnipeg.ca/s/FFJiJMNt9Czgo72` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Quantum Reality or Not**

   To answer this question, you will need to watch the video of Sidney Coleman's famous lecture "Quantum Mechanics In Your Face" at `https://www.youtube.com/watch?v=EtyNMlXN-sw` . (This is part of the reading assignment; the transcript is at `https://arxiv.org/pdf/2011.12671.pdf`.)

   (a) The Bell experiment considers 2 distinguishable spin 1/2 particles in the singlet ($s = 0$) total spin state. If $\hat{a}$ and $\hat{b}$ are two unit vectors, show that

   $$\left\langle \left(\hat{a}\cdot\vec{S}^{(1)}\right)\left(\hat{b}\cdot\vec{S}^{(2)}\right)\right\rangle = -\frac{\hbar^2}{4}\hat{a}\cdot\hat{b}\ . \tag{1}$$

   Hint: Think about a convenient choice of axes and remember that the spin operators are given in matrix form as $S_i \simeq (\hbar/2)\sigma_i$ in terms of the Pauli matrices.

   (b) Three electrons are prepared in the so-called "GHZM" spin state

   $$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3) \tag{2}$$

   described in the video. Show that $|\psi\rangle$ is an eigenstate of the operator $S_x^{(1)}S_y^{(2)}S_y^{(3)}$ and find the eigenvalue.

2. **Interpretations of Quantum Mechanics** *samples from previous midterms*

   Answer the following questions.

   In the first two parts, consider quantum teleportation. Quantum teleportation transfers an unknown state $|\psi\rangle$ from one qubit to another at a distance. This process involves two measurements.

   (a) In the Copenhagen interpretation of quantum mechanics, is quantum teleportation described by a unitary operation? Explain very briefly.

   (b) In the many worlds interpretation of quantum mechanics, is quantum teleportation described by a unitary operation? Explain very briefly.

   For the following, choose the best answer from the options given and write a very brief explanation.

   (c) Which of the following represents the state of the electron-positron pair (particles 1 and 2) and the two observers A and B after all measurements in a many-worlds interpretation of the EPR experiment as described in the lecture notes?
   A. $|\uparrow\rangle_1|\downarrow\rangle_2|\text{sees }\uparrow\rangle_A|\text{sees }\downarrow\rangle_B$ B. $(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)|\text{sees }\uparrow\rangle_A|\text{sees }\downarrow\rangle_B/\sqrt{2}$
   C. $(|\uparrow\rangle_1|\downarrow\rangle_2|\text{sees }\uparrow\rangle_A|\text{sees }\downarrow\rangle_B - |\downarrow\rangle_1|\uparrow\rangle_2|\text{sees }\downarrow\rangle_A|\text{sees }\uparrow\rangle_B)/\sqrt{2}$

(d) If I state that whether Schrödinger's cat lives or dies is predetermined by secret physics of the radioactive nucleus before I close it into the box, what type of theory of quantum mechanics am I expressing?
A. Hidden Variables Theory B. Copenhagen Interpretation C. Many Worlds Theory
D. Bell's Theory

3. **Entanglement Yes/No** *previous midterm sample*

Is each of the following pairs entangled? Answer yes or no and explain very briefly.

(a) The spins of an electron and positron in a total spin $s = 0$ state, as we discussed for the EPR experiment.

(b) Two electrons in an atom in the total angular momentum $|2, 0\rangle$ state, which is written as $|2, 1\rangle = (|1, 1\rangle_1|1, 0\rangle_2 + |1, 0\rangle_1|1, 1\rangle_2)/\sqrt{2}$ in terms of the individual electron orbital angular momenta.

(c) Two qubits, initially in state $|0\rangle|0\rangle$, after application of the Hadamard gate on each qubit followed by the CNOT gate.

4. **Quantum Computing Multiple Choice** *samples from previous midterms*

Chose the correct answer for each part. Explain your answers very briefly.

(a) Consider the NOT gate that acts on one qubit of a quantum computer. Which of the following describes its properties?
A. Unitary, not Hermitian B. Hermitian, not Unitary C. Unitary, Hermitian

(b) Which of the following equals the Hadamard operator $\mathbb{H}$?
A. $|1\rangle\langle 0| + |0\rangle\langle 1|$ B. $|0\rangle\langle 0| - |1\rangle\langle 1|$ C. $|+\rangle\langle +| + |-\rangle\langle -|$ D. $|+\rangle\langle 0| + |-\rangle\langle 1|$