PHYS-4602 Homework 3 Due 31 Jan 2024

This homework is due to `https://uwcloud.uwinnipeg.ca/s/FFJiJMNt9Czgo72` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **3-Particle States** *from some Griffiths problems*

   Consider three particles, each of which is in one of the single-particle states $|\alpha\rangle$, $|\beta\rangle$, or $|\gamma\rangle$, which are orthonormal.

   (a) If the particles are bosons, write down the state where one particle is in each of $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$. *Hint*: This state must be symmetric under the exchange of *any* pair of the bosons.

   (b) Write down all possible 3-particle states (including normalization) with two particles in the same 1-particle state and the third particle in a different 1-particle state, still in the case that the particles are indistinguishable bosons.

   (c) How many linearly independent states can you form if the particles are fermions? Write down all the possible linearly independent states. *Hint*: Similarly to the above, these states must be antisymmetric under the exhange of *any* pair of the fermions.

2. **1-Qbit Density Matrix** *Griffiths & Schroeter 12.6 plus*

   Consider the density matrix ρ for a single qubit (you may consider this to be the spin of a single spin-1/2 particle instead). Here you will prove some properties that generalize to other systems.

   (a) Prove that $\rho^2 = \rho$ if and only if the state is pure. *Hint:* Think about the diagonal form of ρ as a matrix in pure and mixed states.

   (b) Suppose someone hands you a qubit and tells you it is 50% likely to be in either of the states $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. Write the density operator for this qubit as a dyad. Show that it is the same as if the other person told you the qubit was 50% likely to be in either state $|0\rangle$ or $|1\rangle$.

3. **Traces of Operators**

   Use Dirac bra/ket notation and the completeness relation for an orthonormal basis to prove the following (you may assume that the Hilbert space is finite-dimensional):

   (a) $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$ for any two operators A, B

   (b) The expectation value $\langle A\rangle = \mathrm{Tr}(A\rho)$ for any observable A in a mixed state with density operator ρ