Advanced Quantum Mechanics PHYS-4602
In-Class Test

Dr. Andrew Frey

26 Feb 2024

Instructions:

- Do not turn over until instructed.
- You will have 50 minutes to complete this test.
- No electronic devices or hardcopy notes are allowed (except as permitted by the illness policy below).
- If you are ill on the date of the test, you must email me no later than 10:30AM on the test date to receive a non-zero mark. If you feel well enough to take the test at home, you may take the exam remotely if you meet the following conditions: You must log into our class zoom with camera turned on and able to see your torso, arms, head, and workspace. You will have 5 minutes after the conclusion of the test to scan your work and upload it to the assignment upload link. You may use computing resources only for zoom (including to ask me questions via chat or voice) and to upload your test solutions. If you cannot take the exam remotely, I will designate one or more problems on the final exam to count both for your midterm mark and toward your final exam.
- INSTRUCTIONS REGARDING TEST LENGTH WILL GO HERE.
- **Answer all questions briefly and completely.**
- **Only the lined pages of your exam book will be graded. Use the blank pages for scratch work only.**

Useful Formulae:

- Dirac Notation
  - Matrix element in $|e_i\rangle$ basis: $A_{ij} = \langle e_i|A|e_j\rangle$
  - Dyad representation in any basis and eigenbasis

$$A = \sum_{i,j} A_{ij}|e_i\rangle\langle e_j| = \sum_n a_n|a_n\rangle\langle a_n|$$

  - With basis chosen, state is column, operator is matrix

- Time Evolution by Schrödinger Equation

$$i\hbar\frac{d}{dt}|\Psi\rangle = H|\Psi\rangle$$

  - Time-evolution by operator $U(t) = \exp(-iHt/\hbar)$ or by energy eigenstates

$$|\Psi(t)\rangle = \sum_n c_n e^{-iE_nt/\hbar}|E_n\rangle$$

- Measurement
  - Measurement of observable A gives an eigenvalue a of A
  - Probability of observing a is $|\langle a|\psi\rangle|^2$ for e'state $|a\rangle$ and system state $|\psi\rangle$
    If multiple e'states have e'value a, sum over e'states
  - The uncertainty $\sigma_{\mathcal{O}}$ obeys $\sigma_{\mathcal{O}}^2 = \langle \mathcal{O}^2\rangle - \langle \mathcal{O}\rangle^2$
  - Heisenberg Uncertainty Prinicple $\sigma_A\sigma_B \geq (1/2)|\langle [A,B]\rangle|$
  - Copenhagen: wavefunction collapses to $|a\rangle$ on measurement
    Many Worlds: wavefunction entangles with observer
    Hidden Variables: value of measurement determined in advance

- Multiple Degrees of Freedom
  - State is sum of factorized terms $|\Psi\rangle = \sum |\alpha\rangle_1|\beta\rangle_2\cdots|\psi\rangle_N$ for N d.o.f.
  - Inner products work factor by factor

$$({}_1\langle\psi|{}_2\langle\phi|)(|\alpha\rangle_1|\beta\rangle_2) = \langle\psi|\alpha\rangle_1\langle\phi|\beta\rangle_2\ ,\quad {}_1\langle\psi|(|\alpha\rangle_1|\beta\rangle_2) = (\langle\psi|\alpha\rangle_1)|\beta\rangle_2$$

  - Operators act on whole Hilbert space or one subsystem, ie $A_1(|\alpha\rangle_1|\beta\rangle_2) = (A_1|\alpha\rangle_1)|\beta\rangle_2$
  - Identical bosons have symmetric states, identical fermions have antisymmetric states

- Quantum Information
  - Shannon entropy $S = -\sum P\log_2 P$
  - Density operator $\rho = P_1|e_1\rangle\langle e_1| + P_2|e_2\rangle\langle e_2| + \cdots$ (its eigenbasis), $\rho = |\psi\rangle\langle\psi|$ (pure state)
  - Von Neumann entropy $S = -Tr(\rho\log_2\rho)$ =Shannon entropy of probability e'values
  - Trace of operator $Tr(A) = \sum\langle e_i|A|e_i\rangle$ over orthonormal basis $\{|e_i\rangle\}$
  - Partial trace over subsystem 2 with orthonormal basis $\{|e_i\rangle_2\}$

$$Tr_2(A) = \sum {}_2\langle e_i|A|e_i\rangle_2 = A_1 = \text{operator on subsystem 1}$$

- Quantum Computing
  - 1-bit gates $I : (|0\rangle \to |0\rangle, |1\rangle \to |1\rangle); NOT : (|0\rangle \to |1\rangle, |1\rangle \to |0\rangle);$
    $R(\phi) : (|0\rangle \to |0\rangle, |1\rangle \to e^{i\phi}|1\rangle); \mathbb{H} : (|0\rangle \to (|0\rangle + |1\rangle)/\sqrt{2}, |1\rangle \to (|0\rangle - |1\rangle)/\sqrt{2}$
  - 2-bit controlled NOT gate $CNOT(|x\rangle|y\rangle) = |x\rangle|x\oplus y\rangle$, $\oplus$ = addition mod 2