PHYS-3203 Course Project Instructions

This project is worth 15% of your grade for the entire course.

General Instructions

- The project is due at **10:59PM 4 April 2024**.
- Submissions **must** be typed PDF files. They should be prepared with LaTeX or else MS Word (or similar word processor) with an equation editor for mathematics (*please export your Word file to PDF to submit*). Label your filenames with your first initial, last name, and "project" (for example `AFrey_project.pdf`); if you need to break your solution into multiple parts, label them in order with page numbers (`AFrey_project1.pdf`, `AFrey_project2.pdf`, etc). See the homework submission instructions on the course outline.
- Upload your submissions to `https://uwcloud.uwinnipeg.ca/s/Re9qoZBqcD8F5oe` . **This is the same link as for homework.**
- Each project choice requires numerical analysis (ie, computing) in some way, with some projects requiring more than others. You may use computer software (such as python, Maple, etc) to help solve any part of the project, and you **must** attach your code (or worksheet, etc) as an appendix.
- There are 4 possible project choices below; you must confirm your choice of project with me via email by **19 March 2024**. Only one student may choose a given project, and the project will be assigned to the first student who chooses it. You may alternately design your own project but must consult with me on zoom to specify the project (by the same deadline). Confirming your project by the deadline is worth **10% of the project grade**.
- You will submit your project as a written report such as a lab report or essay, using full sentences. You do not need to include every step of mathematics, but you should explain what you are doing and where any equations come from. The marking rubric is below.
- *Please make an appointment to discuss your project with me if you have questions or need help.* Note that, unlike homework assignments, I have not solved all these problems in advance, so it is appropriate to discuss how to work through difficult parts. Learning is more important than doing it all yourself, and some of these are hard problems. If the problem is too difficult, we can discuss modifying it.

Rubric

- **Choice of topic:** Confirming your project by the deadline is worth **10% of the project grade**.
- **Correctness:** Carrying out an accurate and complete analysis of your project question is worth **30% of the project grade**. This will be marked similarly to a test or exam.
- **Written Report:** The written report is worth **60% of the project grade**. It should be typed as described in the General Instructions using full sentences. There should be a

clearly indicated introduction to the problem as well as an explanation of your methods and solution. If you use any references, there must be a bibliography, and any computer code must be included as an appendix. The 60% marks for the written report will be distributed as follows:

1. **Format: 10%** Do you have an identifiable introduction? Are the bibliography and appendix included when needed? Are equations typeset neatly? Are figures used appropriately?
2. **Description: 25%** Is the problem introduced and explained clearly? Is the final result identifiable?
3. **Explanation: 25%** Are your methods, including any approximations, assumptions, or computer use, explained fully and clearly?

Option 1: Symmetric Top and Liouville Theorem

As we've seen, we can write the Hamiltonian for the Euler angle θ of a symmetric top as

$$H = \frac{p_\theta^2}{2I} + \left[\frac{(p_\phi - p_\psi\cos\theta)^2}{2I\sin^2\theta} + MgR\cos\theta\right] \tag{1}$$

(plus a constant) for mass M, moment of inertia I, and distance R from support to center of mass. We also know p_ϕ, p_ψ are constants set by initial conditions. To analyze the nutation (motion of θ), start by finding Hamilton's equations for $\dot{p}_\theta$ and $\dot{\theta}$.

Choose some value for the constants and solve Hamilton's equations numerically for your choice of initial conditions of θ, p_θ extending for at least one oscillation in θ. Plot your solution. Then pick several other similar initial conditions and solve for their time evolution. Then make a plot in the θ, p_θ plane at each of several times with one point for each of your solutions. Can you see that the area spanned by the solutions in the θ, p_θ plane stays roughly constant?

Option 2: Lagrange Points

Consider the restricted 3-body problem as described in the class notes. If the radius of the primary-secondary orbit is a, the frequency of the orbit is $\omega = \sqrt{G(m_1+m_2)/a^3}$ for primary and secondary masses m_1, m_2. Work in an accelerating reference frame that rotates with the primary and secondary (ie, they are at fixed positions). Assume all motion is in the xy plane. Then the primary and secondary are at positions $a_1\hat{x}$ and $-a_2\hat{x}$ with $a_1 = m_2 a/(m_1+m_2)$ and $a_2 = m_1 a/(m_1+m_2)$ respectively. You will identify the Lagrange points, as follows.

In this frame, the force on a tertiary of mass m_3 moving in the same plane is

$$\vec{F} = -Gm_3\left(\frac{m_1}{r_1^3}\vec{r}_1 + \frac{m_2}{r_2^3}\vec{r}_2\right) + m_3\omega^2\vec{r} - 2m_3\vec{\omega}\times\dot{\vec{r}}\ . \tag{2}$$

where $\vec{r}$ is the position of the tertiary and $\vec{r}_1 \equiv \vec{r} - a_1\hat{x}$, $\vec{r}_2 \equiv \vec{r} + a_2\hat{x}$. First, find the potential energy function for a stationary tertiary (ie, corresponding to all terms in the force except for the Coriolis force). Write your answer in terms of the masses and primary-secondary orbit radius.

Make a contour plot of this potential energy as well as a plot for tertiary positions on the x axis for the cases $m_1 = m_2$ and $m_1 \gg m_2$. In both cases, use your plots to argue that there are (unstable) equilibrium points between the primary and secondary as well as outside each of them. These are the L_1, L_2, L_3 Lagrange points.

Finally, consider the two points L_4, L_5 which form equilateral triangles with the primary and secondary. Find their (x, y) positions and show that these points are extrema of the potential energy function you found in 2D. Then solve Newton's 2nd law for the force (2) with initial velocity zero and initial position close to one of the L_4, L_5 points. Is it oscillatory or growing?

Option 3: Double Pendulum

A double pendulum is a pendulum hanging from another pendulum. Find the Lagrangian for this system in terms of θ_1 and θ_2, the angles of each pendulum from the vertical (let θ_1 be the angle of the upper pendulum). Find the Euler-Lagrange equations for the two angles. You will have two coupled nonlinear ordinary differential equations. Talk to me if you need some help figuring out how to set up this problem.

Choose masses and lengths for the two pendulums. Then solve the equations of motion numerically for initial values $\theta_1 = 0$ and $\theta_2 = \pi/20, \pi/10, \pi/5, \pi/3, \pi/2$ and $\dot{\theta}_1 = \dot{\theta}_2 = 0$. (Note that you will have 5 different solutions.) Plot these solutions for several cycles of motion. Take a Fourier transform for θ_1, θ_2 in each solution and plot the frequencies you find in each case. Describe the motion. If you change the initial conditions from the $\theta_2 = \pi/2$ slightly, what happens?

Option 4: Design Your Own Project

You will need to think about a project similar to the other 3 options and discuss it with me.