PHYS-3203 Homework 9 Due 28 March 2024

This homework is due to `https://uwcloud.uwinnipeg.ca/s/Re9qoZBqcD8F5oe` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Boosts and Rotations**

   In matrix form, we can define the boost Λ_{tx} along x and the rotation Λ_{xy} in the xy plane (around the z axis) as follows:

$$\Lambda_{tx}(\phi) = \begin{bmatrix} \cosh\phi & -\sinh\phi & & \\ -\sinh\phi & \cosh\phi & & \\ & & 1 & \\ & & & 1 \end{bmatrix} , \quad \Lambda_{xy}(\theta) = \begin{bmatrix} 1 & & & \\ & \cos\theta & \sin\theta & \\ & -\sin\theta & \cos\theta & \\ & & & 1 \end{bmatrix} . \tag{1}$$

   Empty elements in the matrices above are zero.

   (a) In matrix form, the metric $\eta_{\mu\nu}$ is

$$\eta = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} . \tag{2}$$

   Show that both rotation and boost in (1) satisfy the condition $\eta_{\mu\nu} = \Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}\eta_{\alpha\beta}$, which is $\eta = \Lambda^T\eta\Lambda$ in matrix notation.

   (b) First, write down the Lorentz transformation matrix $\Lambda_{ty}(\phi)$ corresponding to a boost along the y direction by permuting axes. Then show that you can get a boost along y by rotating axes, boosting along x, then rotating back by proving that $\Lambda_{ty}(\phi) = \Lambda_{xy}(-\pi/2)\Lambda_{tx}(\phi)\Lambda_{xy}(\pi/2)$.

2. **Some Scalar Products**

   In some frame, the components of two 4-vectors are

$$a^\mu = (2,0,0,1) \text{ and } b^\mu = (5,4,3,0) . \tag{3}$$

   inspired by a problem in Hartle

   (a) Find a^2, b^2, and $a \cdot b$.

   (b) Does there exist another inertial frame in which the components of a^μ are $(1,0,0,1)$? What about b^μ? Explain your reasoning.

   Now consider lightlike 4-vectors f^μ and g^μ.

   (c) If f^μ and g^μ are orthogonal ($f \cdot g = 0$), prove that they are parallel ($f^\mu \propto g^\mu$).

   (d) Is the 4-vector $f^\mu + g^\mu$ spacelike, timelike, or lightlike? Assume that both $f^0 > 0$ and $g^0 > 0$.