

## PHYS-3203 Homework 6 Due 29 Feb 2024

This homework is due to <https://uwcloud.uwinnipeg.ca/s/Re9qoZBqcD8F5oe> by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

### 1. **Steady Precession of a Top** *related to question from Thornton & Marion*

Consider a symmetric top that is initially spinning vertically  $\theta = 0$ , so the two conserved angular momenta  $p_\phi$  and  $p_\psi$  as defined in the notes equal each other.

- (a) Write the effective potential  $U(\theta)$  for motion in  $\theta$  and expand it to order  $\theta^2$ .
- (b) Based on your expansion above, under what conditions is rotation with  $\theta = 0$  stable? Compare this to our discussion of steady precession in the lecture notes. *Hint:* Is  $\theta = 0$  an extremum? If so, what kind?
- (c) Suppose that  $2MgRI < p_\psi^2 < 4MgRI$ . By considering the value of  $U(\theta)$  at  $\theta = 0$  and  $\theta = \pi/2$  and the result of the previous part, argue that the top nutates between  $\theta = 0$  and another angle  $\theta_1 < \pi/2$ .
- (d) Find  $\dot{\phi}$  and  $\dot{\psi}$  when  $\theta = 0$ . Under what condition does  $\dot{\psi}$  have the opposite sign of  $p_\psi$  for  $\theta = 0$ ? (When this occurs, the top spins in the opposite direction of the precession.)