PHYS-3203 Homework 4 Due 8 Feb 2024

This homework is due to `https://uwcloud.uwinnipeg.ca/s/Re9qoZBqcD8F5oe` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Hamilton and Newton** *based on a question from Thornton & Marion*

   Consider an object moving in three dimensions under the influence of a conservative force with potential energy $V(\vec{x})$. In this problem, use Cartesian coordinates x, y, z.

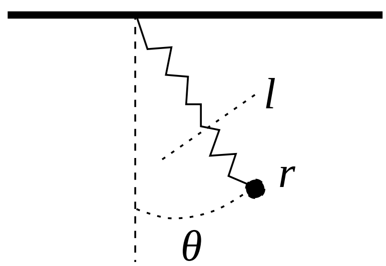
   (a) Find the Lagrangian for this system.

   (b) Find the canonical momentum for each coordinate x, y, z. Then find the Hamiltonian. Note that each canonical momentum is a component of the usual linear momentum and that H is the total energy.

   (c) Find Hamilton's equations and show that they are equivalent to Newton's second law.

2. **Springy Pendulum**

   A mass m hangs from a spring of negligible mass, which in turn hangs from a pivot which allows motion in a plane under the influence of gravity. The spring and mass therefore form a pendulum that can also oscillate radially. The spring has spring constant k and equilibrium length l (in the absence of gravity). See the figure below.



   (a) Find the Lagrangian for this pendulum in terms of plane polar coordinates as shown in the figure above. You can set the gravitational potential energy to be zero at the position of the pivot and the spring potential energy to be zero at $r = l$.

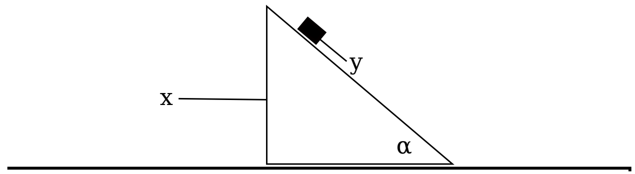
   (b) Find the canonical momenta and Hamiltonian for this system.

   (c) Write Hamilton's equations for this pendulum and find the equilibrium position (the position where all time derivatives vanish).

   (d) Suppose the pendulum starts at rest at position $r = l$, $\theta = \theta_0$. Describe qualitatively the behavior of r as the pendulum swings down to $\theta = 0$ and explain your answer. *Hint:* you can think about what the initial conditions tell us about p_r, p_θ and their time derivatives.

3. **Box on a Wedge Hamiltonian Version**

   Consider again the box of mass m sliding down a wedge of mass M on a frictionless horizontal surface. See the figure



On the previous assignment, you should have found that the Lagrangian is

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 - 2\dot{x}\dot{y}\cos\alpha\right) + mgy\sin\alpha\ . \tag{1}$$

(a) Show that the velocities and canonical momenta are related by

$$p_x = (M+m)\dot{x} - m\dot{y}\cos\alpha\ ,\quad p_y = m(\dot{y} - \dot{x}\cos\alpha) \tag{2}$$

and

$$\dot{x} = \frac{p_x + p_y\cos\alpha}{M + m\sin^2\alpha}\ ,\quad \dot{y} = \frac{p_y}{m} + \frac{p_x + p_y\cos\alpha}{M + m\sin^2\alpha}\cos\alpha\ . \tag{3}$$

(b) Find the Hamiltonian. *Hint:* note that the term in parentheses in the Lagrangian can be written $p_y^2/m^2 + \dot{x}^2\sin^2\alpha$.

(c) Name two conserved quantities in this system.

(d) Find $\dot{p}_y$.