PHYS-3203 Homework 3 Due 1 Feb 2024

This homework is due to `https://uwcloud.uwinnipeg.ca/s/Re9qoZBqcD8F5oe` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Sliding on a Cycloid**

   Consider a cycloidal track (like we found for the brachistochrone)

   $$x = a(\theta - \sin\theta)\ , \quad y = a(1 - \cos\theta) \tag{1}$$

   with y increasing downward. The generalized coordinate θ extends from $\theta = 0$ at the left of the track $x = 0, y = 0$ to $\theta = 2\pi$ at the right $x = 2\pi a, y = 0$, and the lowest point of the track is $x = a\pi, y = 2a$ at $\theta = \pi$.

   To learn more about motion on the cycloid, try a different generalized coordinate. Define

   $$s = \int_{\pi}^{\theta} d\theta' \sqrt{\left(\frac{dx}{d\theta'}\right)^2 + \left(\frac{dy}{d\theta'}\right)^2}\ , \tag{2}$$

   so $|s|$ is the distance traveled from the lowest point along the cycloid.

   (a) Show that the kinetic energy $T = m(\dot{x}^2 + \dot{y}^2)/2$ is $T = m\dot{s}^2/2$ in terms of s. *Hint:* Use the infinitesimal form of the Pythagorean theorem to find $\dot{s}^2$.

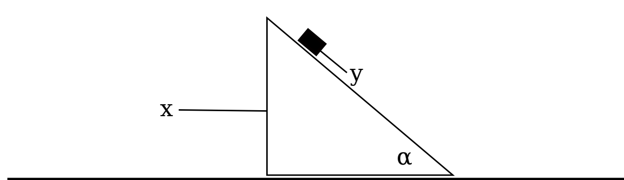
   (b) Write the potential energy $V = -mgy$ in terms of s. *Hint:* Using a half-angle formula, integrate (2) to get s in terms of θ. Then use the angle addition formula to get s^2 in terms of y.

   (c) Using your results for the kinetic and potential energies, write the Lagrangian in terms of s. By comparing the Lagrangian to your result from the Lagrangian of a harmonic oscillator. show that motion on the cycloid is simple harmonic. You do not need to find the Euler-Lagrange equations.

   (d) The previous part proves that the sliding on the cycloid has a frequency (and therefore period) that is independent of amplitude, just like any other harmonic oscillator. What is the frequency of motion?

2. **Box on a Wedge**

   A triangular wedge of mass M is able to slide frictionlessly on a horizontal surface, and a box of mass m can slide frictionlessly down the wedge as in the figure below. The incline of the wedge makes an angle α with the horizontal.

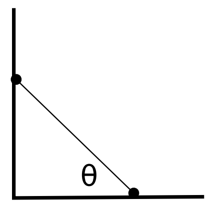
   

   (a) Write the Lagrangian for this system in terms of x, the displacement of the wedge along the horizontal surface, and y, the displacement of the box down the incline. *Hint:* when you find the kinetic energy of the box, start by finding its 2D displacement from its initial position at $x = 0, y = 0$.

(b) Using the Euler-Lagrange equations, find the acceleration of the wedge as the box slides down the incline (as a multiple of the gravitational acceleration g).

3. **Sliding Rod**

   Two point masses of mass m each are attached to the opposite ends of a light rod of length ℓ. The rod leans against a wall at an angle θ from the horizontal, as in the figure below.



   The rod slides down the wall under the influence of gravity.

   (a) Write the Lagrangian for this system in terms of the generalized coordinate θ.

   (b) The rod is initially at rest at angle $\theta = \pi/4$. What is the initial vertical acceleration of the point mass on the wall (what is the second derivative of the Cartesian position of that mass)? *Hint:* recall that $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$.