PHYS-3203 Homework 1 Due 18 Jan 2024

This homework is due to `https://uwcloud.uwinnipeg.ca/s/Re9qoZBqcD8F5oe` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Crossing the Line**

   A light ray travels through a medium with index of refraction n_1 for $x < 0$ and index n_2 for $x > 0$ starting at position $(-X, 0)$, passing through the interface at $(0, y)$, and ending at position (X, Y) for $X \gg Y$. Use Snell's Law to show that the travel time is minimized when $y = n_2 Y/(n_1 + n_2)$. *Hint:* use $X \gg Y$ to argue that the angles of incidence and refraction are small and the fact that $\tan\theta \approx \sin\theta$ for small angles.

2. **A Line Really Is Minimum Length** *from Thornton & Marion and others*

   We know that the minimum length curve in two dimensions that connects the origin to the point $x = y = a$ is the straight line $y(x) = x$. Consider instead the curve $y(x) = x + b\sin(n\pi x/a)$, which also connects the origin to $x = y = a$ if n is an integer.

   (a) Write the length of this curve as an integral over x and, assuming $b \ll a$, expand the integrand to second order in b/a (that is, use a Taylor series for the variable $z = b/a$). *Hint:* use an infinitesimal version of the Pythagorean theorem for the length.

   (b) From your previous result, show that the length of the curve to this order is $\sqrt{2}a + cb^2/a$, where c is a positive number, and find c. This shows that changing the line to a slightly different curve increases the path length. *Hint:* you may find an angle addition formula useful for your integral.

3. **A Sample Euler-Lagrange Equation** *from Cline 5.2*

   Consider the functional

   $$S = \int_{x_1}^{x_2} dx\,\sqrt{x}\sqrt{1 + (y')^2}\ , \tag{1}$$

   which depends on the path $y(x)$.

   (a) Find the Euler-Lagrange equation.

   (b) Solve the Euler-Lagrange equation to find the path $y(x)$ that extremizes S. Show that this path satisfies $x = ay^2 + by + c$ for some constants a, b, c.