# PHYS-3202 Homework 9 Due 30 Nov 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/4tyDmt9EEN2RgCy` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Pendulum in a Bus**

   A pendulum of length $l$ hangs from the ceiling of a bus, which is waiting at a red stoplight (stationary with respect to the earth). At time $t = 0$, the light turns green, and the bus starts driving with constant horizontal acceleration $\vec{a}$. Describe the pendulum's position by $\theta$, the angle from the downward vertical (parallel to the Earth's gravitational acceleration $\vec{g}$).

   (a) Find the equilibrium position $\theta_0$ of the pendulum when the bus is accelerating. Describe how you find your answer from the perspective of both an inertial observer on the ground and an accelerating observer on the bus.

   (b) Describe the motion of the pendulum if it is hanging straight down at the time the bus starts accelerating. Assume $|\vec{a}| \ll g$. *Hint:* start by writing Newton's second law in the $\theta$ direction in the accelerating frame, then show it is approximately the same as for the harmonic oscillator.

2. **The Centrifugal Force is Conservative**

   A particle with position $\vec{r}$ relative to a reference frame rotating with angular velocity $\vec{\omega}$ experiences centrifugal force $\vec{F} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r})$. The reading by Tong claims that this force is conservative with potential energy $V = -m(\vec{\omega} \times \vec{r})^2/2$.

   (a) Use vector product identities to write the potential energy in the form

   $$V = -\frac{m}{2}\left[\vec{\omega}^2\vec{r}^2 - (\vec{\omega} \cdot \vec{r})^2\right] \ . \tag{1}$$

   *Hint:* you can derive this using vector triple product identities, but you can also get this result using the solution to the first problem of homework assignment 1.

   (b) Show that the gradient is

   $$\vec{\nabla}V = -m\left[(\vec{\omega}^2)\vec{r} - (\vec{\omega} \cdot \vec{r})\vec{\omega}\right] \ . \tag{2}$$

   Note that the gradient differentiates only the vector components and not the unit vectors because we are working in the rotating frame. *Hint:* Start by showing that $\vec{\nabla}(\vec{a} \cdot \vec{r}) = \vec{a}$ for any vector $\vec{a}$.

   (c) Finally, use the vector triple product identity to show that the centrifugal force satisfies

   $$\vec{F} = m\left[(\vec{\omega}^2)\vec{r} - (\vec{\omega} \cdot \vec{r})\vec{\omega}\right] = -\vec{\nabla}V \ . \tag{3}$$

3. **Rotating Cylindrical Space Vessel** *inspired by Kibble & Berkshire*

   One possible way residents of a space ship or space station can experience "artificial gravity" is if the vessel rotates around some axis. Consider a cylindrical vessel of radius $R$ rotating with angular frequency $\omega$ around the cylinder axis.

(a) The centrifugal force provides an effective gravitational force for stationary objects. At what height $h$ above the "ground" at radius $R$ a second story in a building have to be in order to have only 90% of the gravitational acceleration?

(b) A train runs around the circumference of the vessel in the direction of the vessel's rotation with speed $v$ relative to the ground. What is the increase in the effective weight (as measured by the normal force) for objects on the train compared to their weight at rest on the ground? Explain your answer first from the rotating frame of the vessel and then from the rotating frame in which the train is at rest. *Hint:* in the frame of the ground, the objects on the train have a centripetal acceleration.