PHYS-3202 Homework 7 Due 16 Nov 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/4tyDmt9EEN2RgCy` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Thin Plate**

   Consider a thin plate of material, which is effectively two-dimensional (this is known as a *lamina*). In this problem, assume that the lamina lies in the $z = 0$ plane. Define I_j as the moment of inertia around an axis passing through the origin parallel to the j axis; that is, I_x is around the x axis, etc.

   (a) *from Taylor* Prove that the moments of inertia $I_z = I_x + I_y$.This is sometimes called the *perpendicular axis theorem.*

   (b) Assume that the lamina is a rectangle of sides with length $2a$ parallel to the x axis and length a parallel to the y axis. The center of the rectangle is at the origin. The lamina has uniform mass surface density and total mass M. Calculate the moments of inertia of this lamina around the x, y, and z axes (running through the origin). You may use the results of part (a) to simplify your calculations.

2. **Octant of a Sphere** *based on Fowles & Cassiday*

   Consider a solid object of uniform density and mass M in the shape of one octant of a solid sphere of radius a. That is, it consist of all points with $r \leq a$ and $x > 0, y > 0, z > 0$.

   (a) Find the density of this object (it is constant) and the center of mass position in Cartesian coordinates. *Hint:* even though you are finding the average Cartesian coordinate, you can integrate in spherical polar coordinates. Also, you can reduce calculations by using the permutation symmetry $x \to y \to z \to x$ of the object.

   (b) Find the moment of inertia for rotations around the z axis passing through origin. *Hint:* Again, the integrals can be carried out in spherical polar coordinates.