PHYS-3202 Homework 5 Due 26 Oct 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/4tyDmt9EEN2RgCy` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Work Done on a Forced Oscillator** *similar to Cline 3.5*

   Consider a harmonic oscillator with damping α and natural frequency ω_0 that experiences a force $F(t) = F\cos(\omega t)$.

   (a) In class, we found the particular solution for x when the driving force is a complex exponential $F(t) = F\exp(i\omega t)$. If that complex solution is $x_1(t)$, show that $x(t) = (x_1(t)+x_1^*(t))/2$ is a solution for the cosine driving force in this problem. What is the particular solution $x(t)$ for the cosine driving force?

   (b) Remember that the work done on an object by a force F (in 1D) is the time integral of $F\dot{x}$. Find the work done on the oscillator by the force $F(t)$ over one period of length $2\pi/\omega$ for the particular solution (which is the late time solution neglecting transients). *Hint:* you will find angle addition formulas to be helpful when doing integrals.

2. **Exponential Forcing**

   Consider a damped harmonic oscillator with natural frequency ω_0 and damping coefficient α. Suppose that it experiences a driving force $F(t) = F\exp(-\beta t)$ (with F, β real). Show that the function $x(t) = A\exp(-\beta t)$ solves the equation of motion and find A (disregard any initial conditions). *Hint:* Newton's second law can be written as

   $$m\ddot{x} + 2m\alpha\dot{x} + m\omega_0^2 x = Fe^{-\beta t} \ . \tag{1}$$

3. **Angular Momentum of a Projectile**

   Consider a projectile of mass m launched from the origin with initial velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$, where y increases upward (y is the vertical coordinate). Calculate the angular momentum of the projectile as a function of time and show that its time derivative is equal to the torque due to gravity. You may assume that the projectile is near the surface of the earth (so the acceleration due to gravity is constant) and air resistance is negligible.