PHYS-3202 Homework 3 Due 5 Oct 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/4tyDmt9EEN2RgCy` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Projectile Motion with Linear Drag**

   In the class notes, we showed that the position of a projectile that experiences gravity and linear drag is

   $$\vec{r}(t) = \frac{m\vec{g}}{\gamma}t + \frac{m}{\gamma}\left(\vec{v}_0 - \frac{m\vec{g}}{\gamma}\right)\left(1 - e^{-\gamma t/m}\right) \tag{1}$$

   as a function of time, where $\vec{v}_0$ is the initial velocity, $\vec{g}$ is the gravitational acceleration, and the drag force is $-\gamma\vec{v}$. The initial position is at the origin.

   (a) Show that this expression gives a parabolic trajectory in the limit of no air resistance $\gamma \to 0$. *Hint:* you will need to take a Taylor expansion of the exponential to evaluate the limit.

   (b) In general, it is not possible to find a closed form solution for the *range*, which is the x position where $z = 0$ at positive time. However, we can approximate it by the limit of x as $t \to \infty$. Find this limit in terms of the x component of the initial velocity.

2. **Circular Orbit**

   A satellite of mass m is in a circular orbit of radius r around a planet of mass M.

   (a) Find the total energy of this orbit.

   (b) The period of an orbit is the amount of time it takes for the satellite to return to its initial position (moving around a full circumference). Find the period.

   (c) Finally, suppose the planet rotates once in a time T. What is the radius of a circular orbit with the same period? This is known as a *synchronous orbit* since the satellite can stay over the same position on the planet.

3. **Dark Matter**

   Consider a particle of mass m at a distance r from the center of a spherically symmetric distribution of mass. In class, we stated that the gravitational force on this particle is $\vec{F} = -GmM(r)\hat{r}/r^2$, where $M(r)$ is the total mass inside the sphere of radius r.

   (a) Suppose that the mass distribution has a uniform density ρ and that the particle is in a circular orbit. What is the particle's velocity? *Hint:* remember that the volume inside a sphere is $4\pi r^3/3$.

   (b) If the velocity of circular orbits is independent of radius, what is the function $M(r)$? This is the measured velocity profile of stars orbiting in galaxies, and it does not agree with the measured mass inside the orbits. This is one piece of evidence for the existence of *dark matter*, a type of matter that does not produce light. Discovering the nature of dark matter is one of the major topics of research in physics today.