PHYS-3202 Homework 2 Due 28 Sept 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/4tyDmt9EEN2RgCy` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Full Circle for a Pendulum**

   Consider a pendulum of length ℓ that makes a complete circle of motion, starting at $\theta = 0$ at the bottom, through $\theta = \pi$ at the top, then back to the bottom.

   (a) At what speed must the pendulum be moving at the top of the circle if the tension vanishes at that point?

   (b) If the tension in the pendulum vanishes at the top of the circle, what are the pendulum's speed and tension at the bottom of the circle?

2. **Double Box Friction**

   A box of mass m rests on top of a box of mass M, which rests on a table. A rope is attached to the box of mass M.

   (a) What are the normal forces acting on the box of mass M?

   (b) With what magnitude tension must I pull the rope to cause the two boxes to start moving together? The coefficient of static friction between the lower box and the table is μ_1 and between the two boxes is μ_2.

   (c) Suppose both boxes are moving with the same velocity relative to the table, and I slowly increase the tension in the rope. At what magnitude of the tension does the upper box start to slip relative to the lower box? The coefficient of kinetic friction μ_k is the same between the lower box and either the table or the upper box.

3. **Turbulent Air Resistance**

   Consider an object falling in a uniform gravitational acceleration g against a quadratic drag force of magnitude ζv^2. In this problem, you will want to recall the hyperbolic trig functions and the relationships $\cosh^2\theta - \sinh^2\theta = 1$, $\tanh\theta = \sinh\theta/\cosh\theta$, $d\cosh\theta/d\theta = \sinh\theta$, and $d\sinh\theta/d\theta = \cosh\theta$.

   (a) Since the object falls straight down, write the velocity vector as $\vec{v} = -v(t)\hat{k}$ and find Newton's second law as a first order differential equation for $v(t)$.

   (b) Show that the speed of the object as a function of time is

   $$v(t) = \sqrt{\frac{mg}{\zeta}}\tanh\left(\sqrt{\frac{\zeta g}{m}}\,t\right) \ , \tag{1}$$

   where m is the object's mass. Assume that $v = 0$ at $t = 0$. Does this formula agree with the terminal velocity from the lecture notes? *Hint:* Integrate Newton's 2nd law by separation of variables and a hyperbolic trig substitution $v = \sqrt{mg/\zeta}\,\tanh\theta$ (and use the identities above).

   (c) Now find the distance traveled as a function of time. Check that your answer has the correct units.