PHYS-3202 Homework 11 NOT DUE

This homework is solely for your use in studying and is *not to be handed in.*

1. **Rotating Lamina**

   A rigid lamina (planar object) has principal moments I_1, I_2, and $I_3 = I_1 + I_2$ (this is the perpendicular axis theorem that we saw on a previous assignment). The components of the angular velocity along the corresponding principal axes are ω_1, ω_2, and ω_3 respectively. Show that $\omega_1^2 + \omega_2^2$ is constant. *Note:* you cannot assume that ω_3 is constant.

2. **Finding Principal Axes**

   Four identical small balls of mass m each are at the following locations in the xy plane: $(x, y, z) = (a, 0, 0), (-a, 0, 0), (a/\sqrt{3}, 2a/\sqrt{3}, 0), (-a/\sqrt{3}, -2a/\sqrt{3}, 0)$. They are held together by very light rods. Treat the balls as idealized point particles and the rods as massless.

   (a) Find the (3D) inertia tensor of this object around the origin.

   (b) Find the principal axes and corresponding moments of inertia for the object.

3. **Euler Angles of a Box**

   In this question, consider a rectangular prism (box) of sides $a \geq b > c$ that rotates as a rigid body. The principle axes $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ are parallel to the sides of length a, b, and c respectively. Initially, $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ are aligned with inertial axes $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ respectively. Choose the correct answer for each part. Explain your answers very briefly. **You may use a small box to visualize the rotations.**

   (a) I rotate the box, so now $\hat{e}_1 = -\hat{k}$, $\hat{e}_2 = \hat{\jmath}$, and $\hat{e}_3 = \hat{\imath}$. Which Euler angles describe this configuration?

   A. $\phi = \pi/2, \theta = 0, \psi = \pi/2$
   B. $\phi = \pi/2, \theta = \pi/2, \psi = \pi/2$
   C. $\phi = -\pi/2, \theta = -\pi/2, \psi = \pi/2$
   D. $\phi = -\pi/2, \theta = \pi/2, \psi = 0$

   (b) I return the box to its initial alignment. Then I rotate it by Euler angles $\phi = \pi$, $\theta = \pi/2$, and $\psi = \pi$. With which inertial axis is $\hat{e}_2$ aligned?

   A. $-\hat{k}$ B. $\hat{\jmath}$ C. $-\hat{\imath}$ D. $\hat{k}$