PHYS-3203 Course Project Instructions

This project is worth 15% of your grade for the course.

General Instructions

- The project is due at **Weds 22 March 2023**. The presentation will be due **in the class lecture period** and the written report will be due **at 10:59PM** by file upload.
- Written reports **must** be typed PDF files. They should be prepared with LaTeX or else MS Word (or similar word processor) with an equation editor for mathematics (*please export your Word file to PDF to submit*). Label your filenames with your first initial, last name, and "project" (for example `AFrey_project.pdf`); if you need to break your solution into multiple parts, label them in order with page numbers (`AFrey_project1.pdf`, `AFrey_project2.pdf`, etc). See the homework submission instructions on the course outline.
- Upload your submissions to `https://uwcloud.uwinnipeg.ca/s/NwC99SeB7qHz9Ky` . **This is the same link as for homework.**
- You may use computer software (such as python, Maple, etc) to help solve any of the questions for the project, but you **must** attach your code (or worksheet, etc) as an appendix.
- There are 4 possible project choices below; you must confirm your choice of project with me via email by **15 March 2023**. Only one student may choose a given project, and the project will be assigned to the first student who chooses it. You may alternately design your own project but must consult with me on zoom to specify the project (by the same deadline); the project question must relate to topics covered in the course outline. Confirming your project by the deadline is worth **10% of the project grade**.
- Your project submission will include two components: an in-class presentation of 8 to 10 minutes in length (followed by a 2 minute question period) and a written report such as a lab report or essay, using full sentences. In either component, you do not need to include every step of mathematics, but you should explain what you are doing and where any equations come from. The grading rubric is below.
- *Please make an appointment to discuss your project with me if you have questions or need help.* It is appropriate to discuss how to work through difficult parts. Learning is more important than doing it all yourself, and some of these are hard problems.
- Illness policy: If I am ill on 22 March, we will postpone all presentations. If you are ill on that date, we will reschedule your presentation to a later class period. However, I will grant extensions for the written report only in extreme circumstances.

Rubric

- **Choice of topic:** Confirming your project by the deadline is worth **10% of the project grade**.
- **Correctness:** Carrying out an accurate and complete analysis of your project question is worth **30% of the project grade**. This will be marked similarly to a test or exam.

- **Presentation:** The in-class presentation will be **30% of the project grade**. The presentation may use the chalkboard or consist of projected slides (in which case I strongly suggest you test your computer with the projector in 2L14 ahead of time). Of the 30% marks for this part, marks will be distributed as follows:
  1. **Content: 10%** Do you completely describe the problem and your solution in 8-10 minutes?
  2. **Clarity: 10%** Aare all your steps well-identified so the class can follow you? Do you answer questions clearly?
  3. **Style: 10%** Is it legible? Do you make use of equations and diagrams as appropriate?
- **Written Report:** The written report is worth **30% of the project grade**. It should be typed as described in the General Instructions using full sentences. There should be a clearly indicated introduction to the problem as well as an explanation of your methods and solution. If you use any references, there must be a bibliography, and any computer code must be included as an appendix. Of the 30% marks for this part, marks will be distributed as follows:
  1. **Format: 10%** Do you have an identifiable introduction? Are the bibliography and appendix included when needed? Are equations typeset neatly? Are figures used appropriately?
  2. **Description: 10%** Is the problem introduced and explained clearly? Is the final result identifiable?
  3. **Explanation: 10%** Are your methods, including any approximations, assumptions, or computer use, explained fully and clearly?

Option 1: Lagrange Points

Consider the restricted 3-body problem as described in the class notes. If the radius of the primary-secondary orbit is a, the frequency of the orbit is $\omega = \sqrt{G(m_1+m_2)/a^3}$ for primary and secondary masses m_1, m_2. Work in an accelerating reference frame that rotates with the primary and secondary (ie, they are at fixed positions). Assume all motion is in the xy plane. Then the primary and secondary are at positions $a_1\hat{x}$ and $-a_2\hat{x}$ with $a_1 = m_2 a/(m_1+m_2)$ and $a_2 = m_1 a/(m_1+m_2)$ respectively. You will identify the Lagrange points, as follows.

In this frame, the force on a tertiary of mass m_3 moving in the same plane is

$$\vec{F} = -Gm_3\left(\frac{m_1}{r_1^3}\vec{r}_1 + \frac{m_2}{r_2^3}\vec{r}_2\right) + m_3\omega^2\vec{r} - 2m_3\vec{\omega}\times\dot{\vec{r}}\ .$$

where $\vec{r}$ is the position of the tertiary and $\vec{r}_1 \equiv \vec{r} - a_1\hat{x}$, $\vec{r}_2 \equiv \vec{r} + a_2\hat{x}$. First, find the potential energy function for a stationary tertiary (ie, corresponding to all terms in the force except for the Coriolis force). Write your answer in terms of the masses and primary-secondary orbit radius.

Sketch the effective potential for tertiary positions on the x axis for the cases $m_1 = m_2$ and $m_1 \gg m_2$. In both cases, use your sketch to argue that there are (unstable) equilibrium points between the primary and secondary as well as outside each of them. These are the L_1, L_2, L_3 Lagrange points. You might find it helpful to make plots using software such as Maple.

Finally, consider the two points L_4, L_5 which form equilateral triangles with the primary and secondary. Find their (x, y) positions and show that these points are extrema of the potential energy function you found in 2D.

Option 2: Flipping Box

This is inspired by a problem from Kibble & Berkshire. This problem requires some knowledge of moments of inertia from PHYS-3202.

Consider a uniform cubic box of mass M and side $2a$ sliding frictionlessly on a surface with velocity $\vec{v} = v\hat{\imath}$. The sides of the cube are oriented in the $\hat{\imath}, \hat{\jmath}, \hat{k}$ directions. The cube hits a barrier at $x = 0$ (extending in y), which stops the front edge instantaneously.

Using angular momentum conservation around the position of the barrier, find the angular velocity of the box around the barrier immediately after the collision. You will need to compare the angular momentum of translational motion before the collision to the rotational motion after. How much kinetic energy does the box lose in the collision?

Find the minimum initial speed the cube needs to flip all the way over after the collision. Assume that the edge of the cube that first hits the barrier stays at $x = 0$ afterwards. Note that the CM of the cube must reach a position directly above the barrier for the box to flip over.

Option 3: Invariance of Light Speed

For this project, you will show that the speed of light is invariant, no matter the direction of the velocity. (Of course, our derivation of the Lorentz transformations showed this for light moving along the relative motion of two frames.) In the S frame, where a light beam leaves the origin at time $t = 0$ and reaches point $\vec{x} = (x_0, y_0, 0)$ at time $t = \sqrt{x_0^2 + y_0^2}/c$, hitting a detector there.

Now consider a frame S' moving at speed v relative to S along the x axis. At what coordinates t', x', y', z' does the light hit the detector? Write your answer in terms of c, x_0, y_0, and v.

Find the components of the light's velocity in the S' frame by dividing x'/t', etc. Verify that the speed of light in S' is c.

However, the velocity of the light ray changes in direction. If we take $x_0 = 0$ (so the beam travels along the y axis in frame S), what angle does the light beam make to the y' axis in frame S'? Show that this agrees with our argument about stellar aberration if $v \ll c$.

Option 4: Tachyons

Lorentz symmetry is actually consistent with particles, called *tachyons*, that travel faster than the speed of light, as you'll now see. For now, start by defining a tachyon as a particle with $|d\vec{x}/dt| > c$.

Show that the invariant interval along a tachyon's worldline is positive, so it represents a proper distance s along the worldline (rather than a proper time). Then define a "tachyonic 4-velocity" $V^\mu = c\,dx^\mu/ds$. Give its components in terms of $\vec{v} \equiv d\vec{x}/dt$. Use this to argue that tachyons cannot move *slower* than light.

Show that $V_\mu V^\mu = +c^2$. Then, suppose the component V^0 is positive in some reference frame. Show that there exist reference frames where V^0 is negative, meaning that the tachyon travels backward in time.

Finally, define the tachyon 4-momentum as $p^\mu = mV^\mu$. Suppose an electron — a *normal* massive particle — with mass M starts at rest and emits a tachyon with tachyonic mass m (so the final state is an electron plus a tachyon). Use conservation of 4-momentum to show that the tachyon has energy $-(m^2/2M)c^2$ and the final electron energy is $(M + m^2/2M)c^2$. This is an example of why tachyons are considered instabilities in particle physics; they can cause "spontaneous acceleration" of particles.

You may need to read ahead in your notes or discuss this project question with me.

Option 5: Design Your Own Project

You will need to think about a project similar to the other 4 options and discuss it with me on zoom.