PHYS-3203 Homework 9 Due 29 Mar 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/NwC99SeB7qHz9Ky` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Cherenkov Radiation and Electron Motion**

   Cherenkov radiation occurs when a charged particle moves through a substance faster than the speed of light in that substance (but slower than c, the speed of light in a vacuum). You may use natural units for your answers.

   (a) The speed of light in certain types of glass is about $3c/5$. What is the minimum energy of an electron that can produce Cherenkov radiation in that glass? The mass of an electron is 0.5 MeV in natural units. Give your answer to one digit in MeV.

   (b) The Cherenkov radiation itself is made of photons emitted by the interactions of the electron with atoms in the substance. Let p^μ be the 4-momentum vector of one Cherenkov radiation photon with energy E'. What is the square of p^μ in the rest frame of the electron from the previous part?

   (c) Suppose an electron with a greater energy E is moving in the $+z$ direction. Write its 4-velocity vector U^μ. *Hint:* start by finding the component U^0 and then use what you know about the square U^2 to find the spatial components.

   (d) Now suppose there are two electrons with energy E as measured in the lab frame. One moves along the $+z$ axis (ie, in the $\hat{k}$ direction) and the other along the $+x$ axis ($\hat{\imath}$ direction). What is the energy of the second electron in the rest frame of the first electron? *Hint:* think about the relativistic dot product of the 4-momentum of one electron with the 4-velocity of the other. What is the 4-velocity in the electron's rest frame?

   (e) Finally, consider the two electrons of the previous part. What is their total energy in their CM frame?

2. **SN1987A and Neutrino Masses**

   On 23 Feb 1987, astronomers were startled by the observation of a new supernova in the Large Magellanic Cloud, a satellite galaxy of our Milky Way. However, the first observation of this supernova was several hours earlier by the detection of neutrinos, which was confirmed by two detectors. (The neutrinos arrived before the light because light is trapped for a while by all the matter inside the exploding star.) The fact that the neutrinos all arrived within a few seconds of each other after traveling for more than 100,000 lightyears allows us to put tight constraints on the mass of the neutrino. This problem will guide you through a real calculation of this limit.

   (a) Show that a neutrino with energy $E \gg mc^2$ has a speed approximately given by

   $$\frac{|\vec{u}|}{c} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 . \tag{1}$$

   Hint: We gave formulas in class for energy both in terms of the spatial momentum and in terms of the speed. Try looking at those. Then you will need to make an expansion in powers of mc^2/E.

(b) Light (once free of the matter in the supernova) takes a time $t_0 = 5.3 \times 10^{12}$ s to travel from SN1987A to the earth. How long would a neutrino of energy E take to reach earth from the supernova? Work to the lowest non-trivial order in mc^2/E and give the answer in terms of t_0, m, c, and E. Use (1).

(c) The Kamioka detector in Japan detected several neutrinos. The first arrived with energy 21.3 MeV, and another with energy 8.9 MeV arrived 0.303 s later. Assuming that the second neutrino left the supernova no more than 1 s before the first, what is the maximum neutrino mass m? For simplicity, we are ignoring the possible error in the measurements. *Hint:* The observation time of each neutrino is its emission time plus its travel time; take the difference of these and be careful of signs.

For your interest, these neutrino measurements were made by a predecessor experiment to one of the experiments that led to the 2015 Nobel Prize in Physics. As it happens, one of the people who first discovered this supernova is from Winnipeg!