PHYS-3203 Homework 8 Due 15 Mar 2023

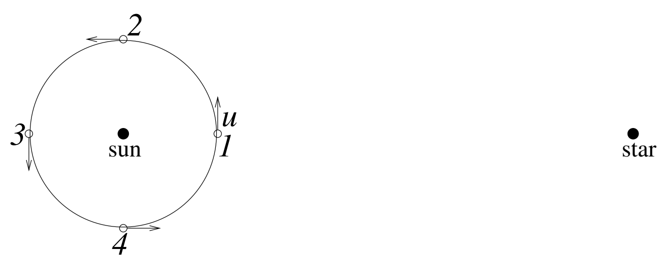
This homework is due to `https://uwcloud.uwinnipeg.ca/s/NwC99SeB7qHz9Ky` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Stellar Aberration**

   In this problem we will explore more the aberration of starlight that was measured as far back as 1725. In this problem, all speeds of objects are small compared to the speed of light, so you are free to use Newtonian/Galilean relativity. You may want to recall that the speed of light is approximately $c = 3 \times 10^8$ m/s.

   (a) First, to get a feel for how this works, consider the following situation. You're driving in a car, and it's raining. Relative to the fixed earth, the rain falls straight down with speed w, and you drive at speed u. At what angle from the vertical do you see the rain falling?

   (b) Now, suppose there is a star straight overhead compared to your telescope. However, the earth is at position 1 in its orbit around the sun (see the figure below), where the orbital speed of the earth is approximately $u = 30,000$ m/s. At what angle from the vertical must I, standing on the earth, aim my telescope so that light from the star falls down the telescope tube? You may ignore the rotational speed of the earth's surface, which is much smaller than the earth's orbital speed. *Hint:* Recall that $\tan\theta \approx \sin\theta \approx \theta$ for small angles θ.



   Note that the figure is not to scale; the star is far enough away that it is effectively directly overhead (at the appropriate time of day) no matter where the earth is in its orbit. Give the angle in arc-seconds, where 3600 arcsec equal 1 degree.

   (c) At which point(s) as labeled in the figure above is this angle of aberration maximized? At which points is it minimized?

2. **Boosts and Rotations**

   In matrix form, we can define the boost Λ_{tx} along x and the rotation Λ_{xy} in the xy plane (around the z axis) as follows:

$$\Lambda_{tx}(\phi) = \begin{bmatrix} \cosh\phi & -\sinh\phi & & \\ -\sinh\phi & \cosh\phi & & \\ & & 1 & \\ & & & 1 \end{bmatrix} , \quad \Lambda_{xy}(\theta) = \begin{bmatrix} 1 & & & \\ & \cos\theta & \sin\theta & \\ & -\sin\theta & \cos\theta & \\ & & & 1 \end{bmatrix} . \tag{1}$$

   Empty elements in the matrices above are zero.

(a) In matrix form, the metric $\eta_{\mu\nu}$ is

$$\eta = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} . \tag{2}$$

Show that both rotation and boost in (1) satisfy the condition $\eta_{\mu\nu} = \Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}\eta_{\alpha\beta}$, which is $\eta = \Lambda^T \eta \Lambda$ in matrix notation.

(b) First, write down the Lorentz transformation matrix $\Lambda_{ty}(\phi)$ corresponding to a boost along the y direction by permuting axes. Then show that you can get a boost along y by rotating axes, boosting along x, then rotating back by proving that $\Lambda_{ty}(\phi) = \Lambda_{xy}(-\pi/2)\Lambda_{tx}(\phi)\Lambda_{xy}(\pi/2)$.

3. **Some Scalar Products**

In some frame, the components of two 4-vectors are

$$a^\mu = (2, 0, 0, 1) \text{ and } b^\mu = (5, 4, 3, 0) . \tag{3}$$

inspired by a problem in Hartle

(a) Find a^2, b^2, and $a \cdot b$.

(b) Does there exist another inertial frame in which the components of a^μ are $(1, 0, 0, 1)$? What about b^μ? Explain your reasoning.

Now consider lightlike 4-vectors f^μ and g^μ.

(c) If f^μ and g^μ are orthogonal ($f \cdot g = 0$), prove that they are parallel ($f^\mu \propto g^\mu$).

(d) Is the 4-vector $f^\mu + g^\mu$ spacelike, timelike, or lightlike? Assume that both $f^0 > 0$ and $g^0 > 0$.