PHYS-3203 Homework 7 Due 8 Mar 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/NwC99SeB7qHz9Ky` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Chain Sliding Off a Table**

   A chain of linear mass density μ and length L lies stretched out linearly on a table with one end hanging over the end. The chain slides off the table frictionlessly, and gravity pulls on the part of the chain hanging off the table. The length of the chain hanging off the table is x, and the chain slides off the table at speed $\dot{x}$.

   (a) What is the vertical center of mass position of the chain, assuming the table is at vertical position $y = 0$?

   (b) What are the kinetic and potential energies of the chain?

   (c) If the chain is initially at rest when $x = 0$ (ie, when it is just starting to slide off the table), use energy conservation to find the speed when $x = L$ (when it has just slide entirely off the table).

2. **Rocket Science**

   Consider a rocket of initial velocity v_0 and initial total mass (including fuel) m_0 moving linearly in outer space. Recall from class that its velocity at a later time t is $v = v_0 + u \ln(m_0/m)$, where u is the exhaust speed relative to the rocket and m is the mass at time t.

   (a) *from Thornton & Marion* What is the ratio m/m_0 when the momentum of the rocket is maximized? *Hint:* Remember that the mass of the rocket is changing as it burns and exhausts fuel.

   (b) *from Cline 2.10* Assume the rocket exhausts fuel at a constant rate $\dot{m} = -k$ (until the fuel runs out). Find the displacement as a function of time.

   (c) Finally, assuming that the relative exhaust speed u is constant, what is the mass of the rocket as a function of time if its acceleration a is constant?

3. **Masses Rotating with a Spring**

   Two point-like objects of identical mass m are held together at a distance a from each other by a compressed spring that has potential energy E. They slide across a frictionless surface and rotate around their center of mass with a period T.

   (a) Show that the relative velocity of the two masses is $2\pi a/T$ in the angular direction around their center of mass and find the initial kinetic energy in their center of mass frame.

   (b) Later, the spring releases all its potential energy, pushing the two masses directly apart (so there is no torque). What is the final relative speed of the two masses?