PHYS-3203 Homework 6 Due 1 Mar 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/NwC99SeB7qHz9Ky` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Exploding Cannonball** *inspired by a problem by Barton (and other texts)*

   A cannonball is launched in an arc with velocity $\vec{u}$. At the top of its trajectory, a chemical charge in it explodes into two parts of masses m_1 and m_2 that separate in the horizontal direction only. The explosion releases energy E, which essentially all goes into the kinetic energy of the cannonball pieces. Show that they are separated by a distance $(u_y/g)\sqrt{2E(m_1+m_2)/m_1m_2}$ when they land, where u_y is the initial vertical component of the velocity.

2. **Sun, Earth, and Moon**

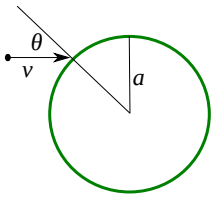
   The earth (mass $M_\oplus$) and moon (mass m) orbit each other at a distance a with period T. The earth-moon center of mass orbits the sun (mass $M_\odot$) at a distance b and period $13T$. You may treat all orbits as circular and use $m \ll M_\oplus \ll M_\odot$.

   (a) What is the total angular momentum in the rest frame of the center of mass of all three objects?

   (b) What is the total kinetic energy in the rest frame of the center of mass of all three objects?

3. **Inelastic Collision with Hoop**

   A point particle and a hoop, both of mass m, move on a frictionless table (the hoop lies flat and slides on the table). The hoop has radius a and therefore moment of inertia ma^2 and is initially at rest. The point particle initially moves at velocity $\vec{v}$ toward the hoop and strikes the hoop with $\vec{v}$ making an angle θ with the vector from the center of the hoop, as in the figure. The point particle sticks to the hoop at that point. There are no external forces.



   (a) What are the initial kinetic energy and angular momentum of the particle and hoop as measured in the rest frame of their center of mass?

   (b) After the collision, the hoop rotates around its center at angular speed ω, so the relative motion of the point particle around the center of the hoop is circular with radius a and frequency ω. Find the angular momentum in the center of mass rest frame in terms of ω and then find ω by using conservation of angular momentum.

   (c) How much kinetic energy is lost during the collision?