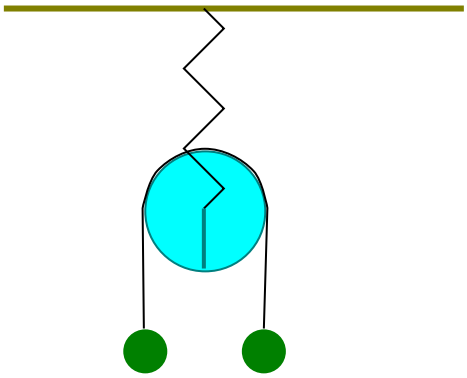
PHYS-3203 Homework 4 Due 1 Feb 2023

This homework is due to `https://uwcloud.uwinnipeg.ca/s/NwC99SeB7qHz9Ky` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Pulley on a Spring**

   Two masses m_1 and m_2 hang on opposite ends of an inelastic string of length L, which itself wraps over a pulley without slipping. The pulley is a uniform disk of mass M, radius R, and moment of inertia $I = MR^2/2$. The center of the disk hangs from a spring of spring constant k and equilibrium length ℓ. There is a line segment marked on the disk along one radius. See the figure.

   (a) Find the Lagrangian for this system in terms of the generalized coordinates y, the distance from the center of the pulley to the ceiling, and θ, the angle of the mark on the disk from the downward vertical. You may leave an undetermined constant in your answer. *Hint:* Remember that the kinetic energy of an object rotating around a fixed axis is $Mv^2/2 + I\omega^2/2$, where v is the velocity of its center of mass and ω is its angular velocity. Then find the distance of each mass m_1 and m_2 from the ceiling.

   (b) Find the Euler-Lagrange equation for the coordinates θ and y

2. **Hamilton and Newton** *based on a question from Thornton & Marion*

   Consider an object moving in three dimensions under the influence of a conservative force with potential energy $V(\vec{x})$. In this problem, use Cartesian coordinates x, y, z.

   (a) Find the Lagrangian for this system.

   (b) Find the canonical momentum for each coordinate x, y, z. Then find the Hamiltonian. Note that each canonical momentum is a component of the usual linear momentum and that H is the total energy.

   (c) Find Hamilton's equations and show that they are equivalent to Newton's second law.