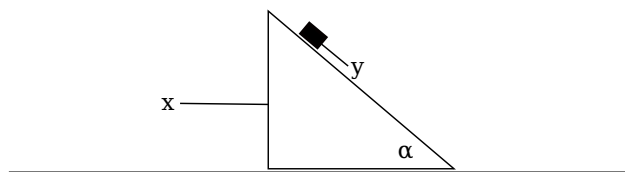


## PHYS-3203 Homework 3 Due 25 Jan 2023

This homework is due to <https://uwcloud.uwinnipeg.ca/s/NwC99SeB7qHz9Ky> by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

### 1. Box on a Wedge

A triangular wedge of mass  $M$  is able to slide frictionlessly on a horizontal surface, and a box of mass  $m$  can slide frictionlessly down the wedge as in the figure below. The incline of the wedge makes an angle  $\alpha$  with the horizontal.



- Write the Lagrangian for this system in terms of  $x$ , the displacement of the wedge along the horizontal surface, and  $y$ , the displacement of the box down the incline. *Hint:* when you find the kinetic energy of the box, start by finding its 2D displacement from its initial position at  $x = 0, y = 0$ .
- Using the Euler-Lagrange equations, find the acceleration of the wedge as the box slides down the incline (as a multiple of the gravitational acceleration  $g$ ).

### 2. Harmonic Oscillator

Consider a simple harmonic oscillator moving in one dimension with restoring force  $F = -kx$ . Write the Lagrangian for the harmonic oscillator and show that the Euler-Lagrange equation is Newton's second law  $m\ddot{x} = -kx$ .

### 3. Sliding on a Cycloid

Consider a cycloidal track (like we found for the brachistochrone)

$$x = a(\theta - \sin \theta) , \quad y = a(1 - \cos \theta) \quad (1)$$

with  $y$  increasing downward. The generalized coordinate  $\theta$  extends from  $\theta = 0$  at the left of the track  $x = 0, y = 0$  to  $\theta = 2\pi$  at the right  $x = 2\pi a, y = 0$ , and the lowest point of the track is  $x = a\pi, y = 2a$  at  $\theta = \pi$ .

To learn more about motion on the cycloid, try a different generalized coordinate. Define

$$s = \int_{\pi}^{\theta} d\theta' \sqrt{\left(\frac{dx}{d\theta'}\right)^2 + \left(\frac{dy}{d\theta'}\right)^2} , \quad (2)$$

so  $|s|$  is the distance traveled from the lowest point along the cycloid.

- Show that the kinetic energy  $T = m(\dot{x}^2 + \dot{y}^2)/2$  is  $T = m\dot{s}^2/2$  in terms of  $s$ . *Hint:* Use the infinitesimal form of the Pythagorean theorem to find  $\dot{s}^2$ .

- (b) Write the potential energy  $V = -mgy$  in terms of  $s$ . *Hint:* Using a half-angle formula, integrate (2) to get  $s$  in terms of  $\theta$ . Then use the angle addition formula to get  $s^2$  in terms of  $y$ .
- (c) Using your results for the kinetic and potential energies, write the Lagrangian in terms of  $s$ . By comparing the Lagrangian to your result from problem 2, show that motion on the cycloid is simple harmonic. You do not need to find the Euler-Lagrange equations. (This actually proves that the sliding on the cycloid has a period that is independent of amplitude, just like any other harmonic oscillator.)